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Synthetic Spectra Method for Carbon Stars

I. The Spectral Synthetic Calculations by Gaussian Quadrature Formula

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炭素星に対するスペクトル合成法

I．ガウスの積分公式による合成スペクトルの計算

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要　旨

スペクトル合成法は，とくに炭素星のように連続スペクトルのレベルの決定が困難な恒星スペクトルの解析に有効な方法である．この方法では，理論的に計算された合成スペクトルを観測されたスペクトルと比較して，恒星大気の物理量や化学組成が求められる．合成スペクトルの計算には大量の計算と記憶容量を要するため，従来この方法は大型計算機を用いてなされており，また，パーソナル・コンピューターを用いる場合には，ミンナートの式によって近似的に計算された合成スペクトルにより比較がなされている．

本研究では，炭素星を対象としたスペクトルの合成法をパーソナル・コンピューターを用いて行う際に，ミンナートの近似式によらないより精度の高い合成スペクトルを計算する方法が試みられた．そのために，筆者らがすでに開発している放射流束のガウス型数値積分のための式を用いて合成スペクトルを計算するプログラムが開発された．そして，辻隆氏の計算した炭素星のモデル大気（$T_{eff}=3400$ K，$g=0.1$ cgs）をもとに，平井正則氏の開発したスペクトル線の波長計算用プログラムを利用して，5885Å～5900Åの波長域のCN，C_2 分子による合成スペクトルが，本プログラムを用いて計算された．PC-9801 VM 2を用いての計算時間は5.5分で，十分実用的であることが示された．なお，本計算では計算を簡便にするために，分子の線のgf値や線輪郭について非現実的な仮定がなされているが，それらの仮定を現実的なものに改めて計算しても，高精度の合成スペクトルが数10分と実用的な時間内で計算されるものと見込まれる．

I. Introduction

Carbon stars are the stars with late spectral type whose spectra are characterized by the predominance of the bands of carbon compounds such as CH, CN and C_2. Dissociative-equilibrium calculations have shown that the spectral characteristics of

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carbon stars result from the peculiar chemical compositions of the atmospheres. Carbon is more abundant than oxygen in the atmospheres of carbon stars, while oxygen is more abundant than carbon in the atmospheres with normal chemical compositions. It is widely believed that the peculiar chemical compositions of the atmospheres of carbon stars result from the mixing of the atmospheres with the inner layers which have undergone nuclear processing, but details of the mixing are not yet elucidated.

Carbon stars are classified into R and N types in the Harvard classification system, which was improved by Shane (1928)[1] who introduced subtypes. It has since become clear that the color temperatures used in the Harvard R-N system do not correspond to the surface temperatures of the atmospheres. Keenan and Morgan (1941)[2] have proposed a new classification system of carbon stars, which is known as the C-classification system. This system is two dimensional and two parameters are used to classify carbon stars. One parameter is expected to indicate the surface temperature and the other one is expected to indicate the carbon abundance. This system was improved by Yamashita (1967)[3] and many carbon stars were classified on this system by him. This system is now most commonly adopted, though some doubt about the validity of this system as a temperature sequence has been expressed by several investigators (for example by Tsuji (1981)[4]).

Formerly, coarse analyses using curve-of-growth have exclusively been done for carbon stars. Recently, fine analyses also have been done for these stars. For carbon stars, synthetic spectra methods rather than curve-of-growth methods have been utilized in fine analyses, because it is difficult to measure accurate values for equivalent widths owing to the uncertainty of continuum. The synthetic spectra methods in fine analyses require a large amount of memory capacity and have been applied only by a large-sized computer. The synthetic spectra methods have been utilized in coarse analyses, too, and have been applied also by a personal-computer[5].

In this paper, a personal-computer program for the synthetic spectra methods in fine analyses of carbon stars has been developed. The divided 4-points Gaussian quadrature formula developed by Nariai and Yoshioka (1983)[6] has been modified and used for the calculation of flux in the present program.

II. The Procedure in the Program

II.1. The Principle of the Synthetic Spectra Methods

In the synthetic spectra methods, spectra of a program star are caluculated assuming physical quantities and chemical composition of the atmosphere and the calculated spectra, i.e. the synthetic spectra are compared with the observed spectra. In case that both of the spectra agree well, the assumed physical quantities and

chemical composition are adopted as those for the program star. Otherwise spectra are synthesized with other physical quantities and chemical composition and are compared with the observed spectra. The process is repeated until an good agreement between the synthetic spectra and the observed ones.

In coarse analyses, the Minnaert Formula (Minnaert (1935)[7]) is usually used for a spectral synthesis calculation, which is

$$\frac{1}{R_\lambda}=\frac{1}{R_c}+\frac{1}{x_\lambda}, \tag{1}$$

where R_λ is the depth (absorbed fraction of continuum radiation) of a line at a wavelength λ, R_c is the limiting central depth of the line, and x_λ is the effective relative optical depth at λ. The effective relative optical depth is the relative optical depth (the optical depth due to the line divided by the optical depth due to the continuum near the line) multiplied by a constant so as to agree with R_λ for weak lines.

This formula holds true for only the Milne-Eddington atmospheric model with a source function depending linearly on the optical depth. If we want to obtain accurately the value of R_λ in general case, we must obtain both the flux of the line at λ and the flux of the continuum near λ and divide the former by the latter. In order to obtain the astrophysical flux F_λ (energy flux divided by π) at a stellar surface, we must calculate the following integral from a model atmosphere :

$$F_\lambda=2\int_0^\infty S_\lambda(\tau_\lambda)\ E_2(\tau_\lambda)\ d\tau_\lambda, \tag{2}$$

where $S_\lambda(\tau_\lambda)$ is the source function of the optical depth τ_λ at λ and $E_2(\tau)$ is the exponential integral function of order 2, which is defined by

$$E_2(\tau)=\int_1^\infty \frac{exp(-\tau t)}{t^2}\ dt. \tag{3}$$

In fine analyses, the above integral is usually calculated with a large-sized computer.

II.2. Calculation of the Integral (2)

In the present program, the integral (2) is calculated on basis of the divided 4-points Gaussian quadrature formula which was developed by Nariai and Yoshioka (1983)[6]. According to this formula, the integral (2) is divided into five subranges of (0, 0.1), (0.1, 0.3), (0.3, 1.0), (1.0, 12.0) and (12.0, ∞), and except for the last subrange (12.0, ∞) the integral for each subrange is calculated by the following 4-points Gaussian quadrature formula for the weight function $E_2(t)$:

$$\int_x^y S(t)\ E_2(t)\ dt=\sum_{i=1}^{4} a_i S(t_i), \tag{4}$$

where a_i's are weights and t_i's are corresponding weighting points. The integral for the last subrange (12.0, ∞) is calculated by the following formula which is derived

on the assumption that $S(\tau)$ is a linear function of τ in this subrange,

$$2\int_{12}^{\infty} S(t)E_2(t)\,dt = \{S(12)-9F\}\{144E_1(12)-11\,exp(-12)\} + 0.5F\{1728E_1(12)-131\,exp(-12)\}, \quad (5)$$

where $E_1(\tau)$ is the exponential integral function of order 1, which is defined by

$$E_1(\tau) = \int_1^{\infty} \frac{exp(-\tau t)}{t}\,dt. \quad (6)$$

Yoshioka and Nariai (1985)[8] modified the above formula in order to improve the accuracy of calculation. As the modification, the subrange of (0, 0.1) was further divided into two subranges of (0, 0.05) and (0.05, 0.1), and the integral for each subrange was also calculated by the 4-points Gaussian quadrature formula for the weight function $E_2(t)$. Although in case of the inner part of atmospheres the modification by Yoshioka and Nariai (1985)[8] improves considerably the accuracy of calculation, it hardly improves in the present case, i.e., in case of the surface of atmospheres. For example, the relative error of the integrated flux at the surface, $\Delta F/F$, for the gray atmosphere calculated by the formula of Nariai and Yoshioka (1983)[6] is equal to 0.001792%, and the corresponding error calculated by the modified formula of Yoshioka and Nariai (1985)[8] is equal to 0.001788%.

In the present program, this formula for the surface are further modified by dividing the subrange of (0, 0.1) into two subranges of (0, 0.01) and (0.01, 0.1) instead of the subranges of (0, 0.05) and (0.05, 0.1), and the integral for each subrange is also calculated by the 4-points Gaussian quadrature formula for the weight function $E_2(t)$. The relative error, $\Delta F/F$, for the gray atmosphere calculated by this modified formula is equal to 0.000418%, which is smaller than those by the formula of Nariai and Yoshioka (1983)[6] and Yoshioka and Nariai (1985)[8] by a factor of about 4. The values of weights and the corresponding weighting points are given in table 1.

The assumption on which the expression (5) is derived holds good for the integrated source function $S(\tau)$ and the mean optical depth τ, but it does not always hold good for the monochromatic source function $S_\lambda(\tau_\lambda)$ and the monochromatic optical depth τ_λ. That is, $S_\lambda(\tau_\lambda)$ cannot always be expressed by a linear function of τ_λ even in a deep layer ($\tau_\lambda > 10$) of the atmosphere, and we cannot always use the expression (5). Furthermore, the value of the integral for this subrange is very small compared with the value of F_λ itself. For example, the relative error of $\Delta F/F$ for the gray atmosphere calculated by the formula of Yoshioka and Nariai (1985)[8], which in this case does not take this subrange into account, is equal to −0.000431%. That is, the ratio of the value of the integral for this subrange to that of the integral for the entire range is equal to only 0.000849% in case of the gray atmosphere. Thus, the integral for this range is neglected in the present program.

Table 1. The values of the weights and the weighting points for the calculation of the flux at the surface by the divided 4-points Gaussian quadrature formula used in the present program.

subrange	weight	weighting point
0～0.01	0.00172099	0.00069017
	0.00318973	0.00328735
	0.00314859	0.00668802
	0.00166413	0.00930234
0.01～0.1	0.014111	0.016061
	0.024890	0.039093
	0.023209	0.069725
	0.011774	0.093589
0.1 ～0.3	0.023299104	0.113252253
	0.039878941	0.163873065
	0.035610382	0.231908553
	0.017461205	0.285510191
0.3 ～1.0	0.046986375	0.342970767
	0.069873539	0.511559098
	0.051680974	0.749107458
	0.021818971	0.945450967
1.0 ～12.0	0.067878833	1.252874435
	0.037638504	2.407230937
	0.004094547	4.740536006
	0.000079668	8.667433737

II.3. Model Atmosphere

The present program is tested with one of the model atmospheres for carbon stars calculated by Querci et al.(1974)[9]. These are the first model atmospheres for carbon stars where the molecular line blanketing effect by CO, CN and C_2 is taken into account through opacity probability distribution functions. In these model atmospheres, it was assumed that the following hypotheses are valid : local thermodynamic equilibrium, hydrostatic equilibrium, plane-parallel layers and steady state. Moreover, the radiative equilibrium was assumed, because the convective flux exists chiefly in the deepest layers for cool atmospheres and has little influence in the regions where molecular lines are formed (e.g., Alexander and Johnson (1972)[10]). The turbulent and radiation pressures were taken into account as well as the gas pressure. The microturbulence velocity was assumed to be equal to 5 km/s throughout the atmosphere.

The relative H/He/C/N/O abundances adopted in the computation were as

follows :

$He/H=0.16$, $C/H=4.1\times10^{-5}$, $N/H=1.48\times10^{-3}$, $O/H=1.25\times10^{-5}$.

The models were computed for the effective temperatures $T_{eff}=3400$, 3800, 4200, 4500 K, and for the surface gravities $g=0.1$, 1.0, 10.0 cgs.

The model used in the present program is the one for $T_{eff}=3400$ K and $g=0.1$ cgs. Table 2 gives details of this model. It gives the optical depth at the standard wavelength ($=0.88\,\mu$) τ_{std}, temperature T, the gas pressure P_g, the radiative flux error $\Delta F/F$, and the depth t in kilometers.

II.4. Wavelengths of CN and C_2 Lines

In the present program, the red system of CN and the Swan system of C_2 are taken into account as the sources of lines. The wavelengths of these lines are calculated according to the program made by Hirai of Fukuoka University of Education, which calculates the wavelengths of CN lines and C_2 lines on the basis of the formulae and molecular constants of Fay, Marenin and Citters (1971)[11] and of Phillips and Davis (1968)[12], respectively. This program calculates the wavelengths of $^{13}C^{14}N$ lines as well as $^{12}C^{14}N$ lines and the wavelengths of $^{13}C^{12}C$ and $^{13}C^{13}C$ lines as well as $^{12}C^{12}C$ lines. Some errors are found in the original program and they are corrected in the present program.

II.5. Other Assumptions Contained in the Present Program

In the present program, the following assumptions are further contained.

a) All the $^{13}C^{14}N$ lines have the same gf value and all the $^{12}C^{14}N$ lines have the same gf value. The ratio of the gf value for $^{12}C^{14}N$ lines to that for $^{13}C^{14}N$ is equal to 89, which value is equal to the solar $^{12}C/^{13}C$ ratio. The same is true for the C_2 lines. That is, the ratio of the gf value for $^{12}C^{12}C$ lines to that for $^{13}C^{12}C$ lines is equal to 89, and the ratio between $^{12}C^{12}C$ lines and $^{13}C^{13}C$ lines is equal to 89×89. Furthermore, the gf value for the $^{12}C^{14}N$ lines is equal to that for the $^{12}C^{12}C$ lines.

b) The ratio of the line absorption coefficient to the continuous absorption coefficient at a given wavelength is constant with optical depth. That is, the changes in dissociation and excitation with the optical depth are negligible. Furthermore, the continuous absorption coefficient is independent of wavelength.

c) The shape of the line absorption coefficient is a isosceles triangle where the vertex corresponds to the line center. The altitude of the triangle is in proportion to the gf value of the line. The widths of the triangle are the same for all the lines.

d) The line absorption occurs according to the mechanism of pure absorption.

e) The atmosphere is in local thermodynamic equilibrium. That is, according to this assumption together with the assumption d), the following relation holds for the source function $S_\lambda(\tau_\lambda)$:

Table 2. The model atmosphere of the carbon star for $T_{\rm eff}=3400$ K and $g=0.1$ cgs calculated by Querci et al. (1974). As for the meaning of the symbol for each column see the text.

$\log_{10}\tau_{\rm std}$	T (K)	$\log_{10}P_{\rm g}$ (dyne/cm^2)	$\Delta F/F$ (%)	t (km)
−6.0	1920	−3.32	−0.11	0
−5.8	1934	−3.12	−0.11	1.71×10^{7}
−5.6	1942	−2.92	−0.10	3.41×10^{7}
−5.4	1956	−2.72	−0.10	5.11×10^{7}
−5.2	1974	−2.57	−0.10	6.81×10^{7}
−5.0	1999	−2.31	−0.10	8.52×10^{7}
−4.8	2030	−2.11	−0.10	1.02×10^{8}
−4.6	2070	−1.90	−0.10	1.20×10^{8}
−4.4	2113	−1.70	−0.03	1.37×10^{8}
−4.2	2158	−1.49	−0.10	1.54×10^{8}
−4.0	2203	−1.28	−0.10	1.72×10^{8}
−3.8	2252	−1.08	−0.10	1.90×10^{8}
−3.6	2303	−0.87	−0.09	2.07×10^{8}
−3.4	2357	−0.67	−0.09	2.25×10^{8}
−3.2	2415	−0.47	−0.09	2.43×10^{8}
−3.0	2476	−0.26	−0.10	2.61×10^{8}
−2.8	2540	−0.06	−0.09	2.78×10^{8}
−2.6	2610	0.13	−0.09	2.96×10^{8}
−2.4	2689	0.33	−0.06	3.13×10^{8}
−2.2	2777	0.51	−0.02	3.30×10^{8}
−2.0	2872	0.69	0.03	3.46×10^{8}
−1.8	2969	0.85	0.01	3.61×10^{8}
−1.6	3071	1.00	−0.05	3.75×10^{8}
−1.4	3174	1.14	−0.07	3.88×10^{8}
−1.2	3279	1.27	−0.13	4.00×10^{8}
−1.0	3388	1.39	−0.26	4.12×10^{8}
−0.8	3502	1.51	−0.46	4.23×10^{8}
−0.6	3624	1.63	−0.49	4.35×10^{8}
−0.4	3750	1.74	−0.33	4.47×10^{8}
−0.2	3887	1.86	−0.34	4.59×10^{8}
0.0	4039	1.99	−0.27	4.72×10^{8}
0.2	4208	2.11	−0.22	4.85×10^{8}
0.4	4424	2.25	−0.11	4.99×10^{8}
0.6	4718	2.38	−0.11	5.13×10^{8}
0.8	5103	2.48	−0.07	5.25×10^{8}
1.0	5579	2.55	−0.06	5.31×10^{8}
1.2	6144	2.59	−0.01	5.34×10^{8}
1.4	6761	2.62	−0.02	5.35×10^{8}
1.6	7397	2.64	0.13	5.36×10^{8}
1.8	8073	2.66	0.82	5.36×10^{8}

$$S_\lambda(\tau_\lambda)=B_\lambda(T), \qquad (7)$$

where $B_\lambda(T)$ is the Planck function for the temperature T corresponding to the optical depth τ_λ.

The above assumptions are made in order to simplify the spectral synthesis calculation, and they do not hold good in real atmospheres. Especially, the assumptions a), b), and c) do not hold at all. However, these assumptions are made, because the main purpose of the present calculation is not to obtain the synthetic spectrum corresponding to a real atmosphere but only to test the effectiveness of the present procedure of calculation.

III. Results and Discussion

On the basis of the above assumptions the synthetic spectrum is calculated for the model atmosphere listed in the table 2 in the following way. First, for each wavelength, the flux without line, i.e., the flux for the continuum $F_{c,\lambda}$ is calculated. Next, the flux with line $F_{l,\lambda}$ for the wavelength is calculated. Then, $F_{l,\lambda}$ is divided by $F_{c,\lambda}$, which value is equal to $1-R_\lambda$ and from which the synthetic spectrum, i.e., the relationship between R_λ and λ is obtained. In order to calculate $F_{c,\lambda}$ and $F_{l,\lambda}$ according to the formula (4), we must know the value of $S_\lambda(t_i)$ corresponding to the weighting point t_i, which value is equal to $B_\lambda(T)$ for T corresponding to t_i according to the equation (7). The value of T corresponding to t_i is obtained via the relationship between $\log_{10}B_{std}(T)$ and $\log_{10}\tau_{std}$, where $B_{std}(T)$ is the Planck function at the standard wavelength ($=0.88\,\mu$). This relationship is used, because $\log_{10}B_{std}(T)$ varies approximately linearly with $\log_{10}\tau_{std}$. The calculation of the optical depth from the absorption coefficient is made by the use of divided 6-points Gauss-Legendre's formula, i.e., the range of integration is divided into subranges and the integral for each subrange is calculated by the 6-points Gauss-Legendre's formula.

The present program is written in BASIC and is named "FLUX". It is listed in the Appendix. Figure 1 shows the synthetic spectrum calculated with this program. The wavelength ranges from 5885 Å to 5900 Å. The step in the calculation is 0.1 Å. For the sake of simpicity in the appearance of the spectrum, only the P_1 branches of the (11, 5) band of the CN red system (9 lines) and the P_1 and R_2 branches of (1, 3) band of the C_2 Swan system (14 lines) are taken into account. The ratio of the line absorption coefficient for $^{13}C^{14}N$ and $^{12}C^{12}C$ to the continuous absorption coefficient at the line center is taken to be 25. The width of the line is taken to be 1.0 Å.

A personal-computer PC-9801 VM 2 (NEC) with double precision floating numbers is used throughout the calculation. In the actual calculation, the program is compiled and is run with V 33 super charger mounted on the slot of the computer for extension of function. The compilation together with the use of V 33 super charger

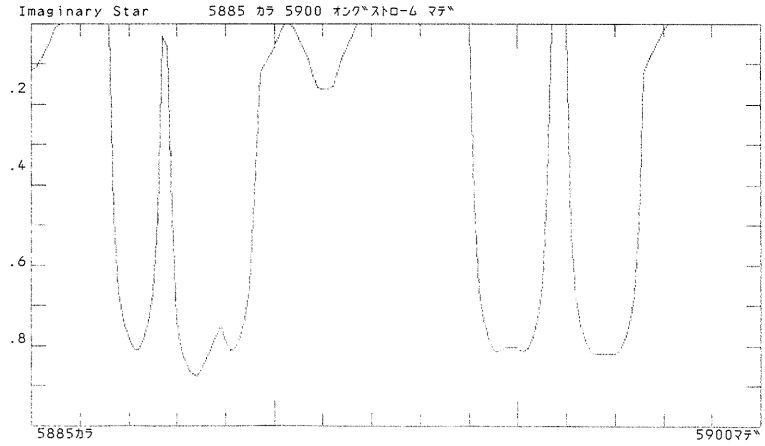

Fig. 1. The synthetic spectrum calculated with the present program, where the flux is calculated by the divided 4-points Gaussian Quadrature formula.

shortens the time of the calculation by a factor of more than 10. For example, It takes 5 minutes and 28 seconds to calculate the spectrum in figure 1 with the compilation and the V 33 super charger, while it takes 58 minutes and 48 seconds to calculate the same spectrum without them.

For the sake of comparison, figure 2 shows the synthetic spectrum for the same range of wavelength as figure 1 calculated with the Minnaert formula (equation(1)). The constant for the effective optical depth is taken to be 1. It takes 2 minutes and 14 seconds to calculate the spectrum of this figure with the compilation and the V 33

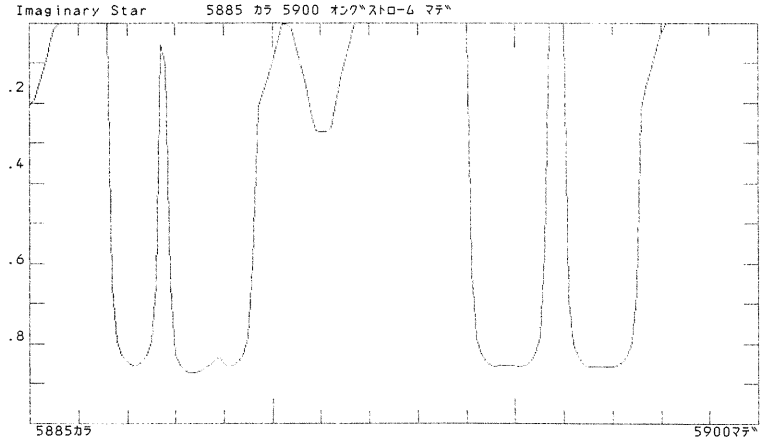

Fig. 2. The synthetic spectrum for the same range of wavelength as the figure 1 calculated with the Minnaert formula.

super charger. The R_c value is taken to be 0.89 in order that the depths of the most strong lines in the two figures should agree. It is noticeable that the depths of weak lines in the figure 2 are deeper than those in the figure 1 and the profiles of strong lines in the figure 2 are more rounded than those in figure 1.

The results obtained in the present study suggests the prospects of calculating the synthetic spectrum accurately without the Minnaert formula by using a personal-computer. The present results are obtained with the assumptions described in the section 2.5. For Comparison with an actual spectrum, it is necessary to replace especially the assumptions a), b), and c) by the conditions corresponding to the actual situations. Although the consideration of these conditions lengthens the time of the calculation, it will not lengthen by a factor of more than 10, and the present procedure of calculation will remain effective. On the basis of the above results, the improvements of the present program are being made by the author.

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Appendix. List of the Program "FLUX"

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10 REM Calculation of Relative Flux at the Surface by the Quadrature Formula of Nariai and Yoshioka
20 REM Calculation of Source Function from the Table of T vs logτ(standard)
30 REM T Value for Given logτ Is Interpolated Via Planck Function at 0.88μ
40 REM τ(at 0.88μ) corresponding to τ(line) is calculated by Gauss's quadrature formula
50 REM by QQT Model:Te=3400K, g=0.1cgs
60 DEFDBL A-H,L-Z:DEFINT I-K
70 DIM AW(20),DP(20),S(20),FL(2)
80 DIM LGTAU(41),T(41),TA(41),DP1(3),DP2(6),BW(3),DPL(30),TAUL(30),DP3(20),LR(20),AR(6)
90 DIM ABSL(100,1),VCN(10,1),A$(10,10),ITS(13,2)
100 C=.5772156649015329#:E1=4.751080258825322D-07:LN=LOG(10#)
110 FOR I=1 TO 20:READ AW(I):NEXT I
120 FOR I=1 TO 20:READ DP(I):NEXT I
130 DATA 0.00172099#,0.00318973#,0.00314859#,0.00166413#,0.014111#,0.024890#,0.023209#,0.011774#
140 DATA 0.023299104#,0.039878941#,0.035610382#,0.017461205#,0.046986375#,0.069873539#,0.051680974#,0.021808971#
150 DATA 0.067878833#,0.037638504#,0.004094547#,0.000079668#
160 DATA 0.00069017#,0.00328735#,0.00668802#,0.00930234#,0.016061#,0.039093#,0.069725#,0.093589#
170 DATA 0.113252253#,0.163873065#,0.231908553#,0.285510191#,0.342970767#,0.511559098#,0.749107458#,0.945450967#
180 DATA 1.252874435#,2.407230937#,4.740536006#,8.667433737#
190 LGTAU(0)=-8#:LGTAU(1)=-7#
200 FOR I=2 TO 41:LGTAU(I)=-6#+.2#*(I-2):NEXT I
210 T(0)=1877#:T(1)=1889#
220 FOR I=2 TO 41:READ T(I):NEXT I
230 DP(0)=0#:DPL(0)=0#
240 FOR I=1 TO 3:READ DP1(I):NEXT I
250 FOR I=1 TO 3:READ BW(I):NEXT I
260 DATA 1920#,1934#,1942#,1956#,1974#,1999#,2030#,2070#,2113#,2158#
270 DATA 2203#,2252#,2303#,2357#,2415#,2476#,2540#,2610#,2689#,2777#
280 DATA 2872#,2969#,3071#,3174#,3279#,3388#,3502#,3624#,3750#,3887#
290 DATA 4039#,4208#,4424#,4718#,5103#,5579#,6144#,6761#,7397#,8073#
300 DATA 0.9324695142031 52#,0.6612093864662645#,0.2386191860831969#
310 DATA 0.1713244923791703#,0.3607615730481386#,0.467913934572691#
320 FOR I=0 TO 41:TA(I)=16349.64#/T(I):TA(I)=-LOG(EXP(TA(I))-1#):NEXT I
330 RO(0)=1:RO(1)=.979002#
340 FOR I=0 TO 0
350 FOR J=0 TO 1:READ VCN(I,J):NEXT J
360 NEXT I
370 FOR I=0 TO 0
380 FOR J=0 TO 1:READ A$(I,J):NEXT J
390 NEXT I
400 DATA 11#,5#
410 DATA P1,R1
420 INPUT "Wavelength(Shortest,Longest,Step,Mark)";LAMDAS,LAMDAL,DLAMDA,MLAMDA
430 LAMDA=LAMDAS:I2=1:I3=0:I4=1
440 GOSUB *CN
450 GOSUB *C2
460 GOSUB *SORT
470 I1=CINT(LAMDAL-LAMDAS)/DLAMDA+1
480 DIM LENG(I1),FLU(I1)
490 WHILE LAMDA<=LAMDAL+.1#*DLAMDA
500 GOSUB *CALCULATION
510 LENG(I2)=LAMDA:FLU(I2)=XF
520 LAMDA=LAMDAS+I2*DLAMDA:I2=I2+1
530 WEND
540 BEEP
550 INPUT "Do You Want to Show the Result at the Display ? Y or N";ANS$
560 IF ANS$="Y" THEN GOSUB *GRAPH1
570 END
580 *CALCULATION
590 FOR I0=1 TO 2
600 GOSUB *SOURCE
610 XF=0#
620 FOR I=1 TO 20:XF=XF+AW(I)*S(I):NEXT I
630 FL(I0)=1.5#*XF
640 NEXT I0
650 XF=FL(2)/FL(1)
660 RETURN
670 *GRAPH1
680 CONSOLE 0,25,0,0:CLS 3:SCREEN 2,0,0,1:COLOR 0
690 DA=(LAMDAL-LAMDAS)/50
700 WINDOW(LAMDAS-DA*2.3,-.05)-(LAMDAL,1.1):VIEW(0,0)-(639,385)
710 INPUT "Name of the Star";NAME$
720 A$="ｶﾗ"
730 LOCATE 2,2
740 PRINT NAME$;SPC(5);LAMDAS;A$;LAMDAL;"ｵﾝｸﾞｽﾄﾛｰﾑ ﾏﾃﾞ"
750 FOR I=1 TO 4
760 LOCATE 0,(I-1)*4.3+6:PRINT .2*I
770 NEXT I
780 B$=STR$(LAMDAS)+"ｶﾗ":C$=STR$(LAMDAL)+"ﾏﾃﾞ"
790 LOCATE 0,25:PRINT SPC(3);B$;SPC(62);C$
800 LINE(LAMDAS,1)-(LAMDAL,0),,B
810 FOR I=1 TO 9
820 LINE(LAMDAS,I/10)-(LAMDAS+DA,I/10):LINE(LAMDAL,I/10)-(LAMDAL-DA,I/10)
830 NEXT I
840 DB=LAMDAS+1
850 WHILE DB<LAMDAL
860 IF DB MOD MLAMDA=0 GOTO 880
```

```
870 LINE(DB,1)-(DB,.97):LINE(DB,0)-(DB,.03):GOTO 890
880 LINE(DB,1)-(DB,.94):LINE(DB,0)-(DB,.06)
890 DB=DB+1
900 WEND
910 FOR I=1 TO I2-2
920 LINE(LENG(I),1#-FLU(I))-(LENG(I+1),1#-FLU(I+1))
930 NEXT I
940 ANS$=INKEY$
950 IF ANS$="" GOTO 940
960 CLS 3:LOCATE 0,0,1:CONSOLE ,,1
970 RETURN
980 *SOURCE
990 TAU=.1#:GOSUB *ABS.RATIO
1000 FOR I=1 TO 20:DPL(I)=DP(I)/RR:NEXT I
1010 TAUL(0)=0#:TAU=DPL(1):GOSUB *ABS.RATIO:TAUL(1)=RR*DPL(1):I=1
1020 WHILE TAUL(I)<DP(20)
1030 IF I=20 GOTO 1110
1040 I=I+1:A=(DPL(I)-DPL(I-1))/2#:B=(DPL(I)+DPL(I-1))/2#:TAUL(I)=TAUL(I-1)
1050 FOR J=1 TO 3:DP2(J)=-A*DP1(J)+B:DP2(7-J)=A*DP1(J)+B:NEXT J
1060 FOR J=1 TO 6:TAU=DP2(J):GOSUB *ABS.RATIO:AR(J)=A*RR:NEXT J
1070 FOR J=1 TO 3:TAUL(I)=TAUL(I)+AR(J)*BW(J):NEXT J
1080 FOR J=4 TO 6:TAUL(I)=TAUL(I)+AR(J)*BW(7-J):NEXT J
1090 WEND
1100 GOTO 1190
1110 TAU=DPL(20):GOSUB *ABS.RATIO:DTAU=(DP(20)-DP(19))/RR:A=DTAU/2#
1120 WHILE TAUL(I)<DP(20)
1130 I=I+1:B=DPL(20)+DTAU*(I-20.5#):DPL(I)=DPL(I-1)+DTAU:TAUL(I)=TAUL(I-1)
1140 FOR J=1 TO 3:DP2(J)=-A*DP1(J)+B:DP2(7-J)=A*DP1(J)+B:NEXT J
1150 FOR J=1 TO 6:TAU=DP2(J):GOSUB *ABS.RATIO:AR(J)=A*RR:NEXT J
1160 FOR J=1 TO 3:TAUL(I)=TAUL(I)+AR(J)*BW(J):NEXT J
1170 FOR J=4 TO 6:TAUL(I)=TAUL(I)+AR(J)*BW(7-J):NEXT J
1180 WEND
1190 II=I
1200 FOR I=1 TO 20
1210 DP3(I)=0#
1220 FOR J=1 TO II
1230 IF DP(I)<TAUL(J) GOTO 1250
1240 NEXT J
1250 IF J=1 GOTO 1350
1260 IF J=II GOTO 1430
1270 FOR K=-2 TO 1
1280 AA=1#:BB=1#
1290 FOR JJ=-2 TO 1
1300 IF JJ<>K THEN AA=AA*(DP(I)-TAUL(J+JJ)):BB=BB*(TAUL(J+K)-TAUL(J+JJ))
1310 NEXT JJ
1320 DP3(I)=DP3(I)+DPL(J+K)*AA/BB
1330 NEXT K
1340 GOTO 1500
1350 FOR K=-1 TO 1
1360 AA=1#:BB=1#
1370 FOR JJ=-1 TO 1
1380 IF JJ<>K THEN AA=AA*(DP(I)-TAUL(1+JJ)):BB=BB*(TAUL(1+K)-TAUL(1+JJ))
1390 NEXT JJ
1400 DP3(I)=DP3(I)+DPL(1+K)*AA/BB
1410 NEXT K
1420 GOTO 1500
1430 FOR K=-2 TO 0
1440 AA=1#:BB=1#
1450 FOR JJ=-2 TO 0
1460 IF JJ<>K THEN AA=AA*(DP(I)-TAUL(II+JJ)):BB=BB*(TAUL(II+K)-TAUL(II+JJ))
1470 NEXT JJ
1480 DP3(I)=DP3(I)+DPL(II+K)*AA/BB
1490 NEXT K
1500 NEXT I
1510 FOR I=1 TO 20
1520 LGTAUU=LOG(DP3(I))/LN
1530 FOR J=0 TO 41
1540 IF LGTAUU<LGTAU(J) GOTO 1560
1550 NEXT J
1560 V=5#*(LGTAUU-LGTAU(J-1))-.5#:D1Y=TA(J)-TA(J-1):D2Y=TA(J-2)-TA(J-1)-TA(J)+TA(J+2)
1570 D3Y=-TA(J-2)+3#*TA(J-1)-3#*TA(J)+TA(J+1)
1580 TTA=(TA(J-1)+TA(J))/2#+V*D1Y+(V^2-.25#)*D2Y/4#+V*(V^2-.25#)*D3Y/6#
1590 TTA=16349.64#/LOG(EXP(-TTA)+1#)
1600 S(I)=1#/(EXP(16349.64#*8800#/LAMDA/TTA)-1#)
1610 NEXT I
1620 RETURN
1630 *ABS.RATIO
1640 IF I0=1 THEN RR=1#:RETURN
1650 RR=1#:LAMDASS=LAMDA-.5#:LAMDALL=LAMDA+.5#:I5=I4
1660 WHILE ABSL(I5,0)<=LAMDALL AND I5<=I3
1670 IF ABSL(I5,0)<LAMDASS THEN I4=I5+1:GOTO 1690
1680 RR=RR+25#*ABS!(I5,1)*(1#-ABS((ABSL(I5,0)-LAMDA)/.5#))
1690 I5=I5+1:WEND
1700 RETURN
1710 *CN
1720 REM CN LINE CALCULATION
1730 GOSUB *DAT
1740 FOR IAA=0 TO 0:V1=VCN(IAA,0):V2=VCN(IAA,1)
1750 FOR IAAA=0 TO 1:A$=A$(IAA,IAAA)
```

```
1760 FOR IA=0 TO 1
1770 RO=RO(IA):GF=1#/89#^IA
1780 FOR IAAAA=0 TO 80:NA=IAAAA
1790 GOSUB *ASG
1800 GOSUB *GVU:GOSUB *FCU:GOSUB *FJU:GOSUB *GEU
1810 GOSUB *GVL:GOSUB *FCL:GOSUB *FJL:GOSUB *GEL
1820 YY=(GVU+GEU+FJU)-(GVL+GEL+FJL)
1830 DNN=.0002871#*(1+.00567#/(YY*100000^2)
1840 LAMD=1/YY-1/YY*DNN:LAMD=LAMD*100000000#
1850 IF LAMD<LAMDAS OR LAMD>LAMDAL GOTO 1870
1860 I3=I3+1:ABSL(I3,0)=LAMD:ABSL(I3,1)=1#*GF
1870 NEXT IAAAA,IA,IAAA,IAA
1880 RETURN
1890 REM  MOLECULAR CONSTANT
1900 *DAT
1910 FOR IB=0 TO 4 :READ OMGU(IB):NEXT
1920 FOR IB=0 TO 4 :READ OMGL(IB):NEXT
1930 FOR IB=0 TO 7 :READ BU(IB)  :NEXT
1940 FOR IB=0 TO 5 :READ BL(IB)  :NEXT
1950 FOR IB=0 TO 3 :READ DU(IB)  :NEXT
1960 FOR IB=0 TO 3 :READ DL(IB)  :NEXT
1970 FOR IB=0 TO 1 :READ HU(IB)  :NEXT
1980 FOR IB=0 TO 1 :READ HL(IB)  :NEXT
1990 FOR IB=0 TO 2 :READ A0(IB)   :NEXT
2000 FOR IB=0 TO 2 :READ P0(IB)   :NEXT
2010 FOR IB=0 TO 2 :READ Q0(IB)   :NEXT
2020 FOR IB=0 TO 3 :READ GMA(IB) :NEXT
2030 FOR IB=0 TO 5 :READ W0(IB)  :NEXT
2040 FOR IB=0 TO 5 :READ W1(IB)  :NEXT
2050 DATA 9240.041#,1813.474#,-12.8272#,5.610D-3,-4.192D-4
2060 DATA 0,2068.435#,-12.9765#,-3.082D-2,1.228D-3
2070 DATA 1.71547#,-1.73452D-2,9.583D-6,-2.756D-6
2080 DATA 4.323D-8,-3.324D-9,1.6D-10,3D-12
2090 DATA 1.89931#,-1.72786D-2,-4.740D-5,4.512D-7,3.533D-10,7.87D-12
2100 DATA 6.1534D-6,7.81D-9,6.83D-10,1.164D-10
2110 DATA 6.3782D-6,4.39D-8,-9.65D-9,6.9D-10
2120 DATA 7.47D-12,-8.0D-14
2130 DATA 8.07D-12,-5.0D-14
2140 DATA -52.694#,8.4652D-2,3.5089D-3
2150 DATA -6.4986D-3,1.6160D-4,-9.3257D-5
2160 DATA 3.3115D-4,9.470D-7,2.332D-6
2170 DATA 1.5687D-2,-2.835D-4,1.9084D-4,-1.9352D-5
2180 DATA 0,6.07#,0.033#,-1.19D-2,-0.461#,3.8D-4
2190 DATA 0,0.08#,-0.01#,1.4D-3,0.1#,-3.1D-4
2200 RETURN
2210 REM *********** ASSIGNMENT ******************
2220 *ASG
2230 IF A$="P1"  THEN LU=-1:LL=-1:JA=NA-.5# :RETURN
2240 IF A$="Q1"  THEN LU=-1:LL=-1:JA=NA+.5# :RETURN
2250 IF A$="R1"  THEN LU=-1:LL=-1:JA=NA+1.5#:RETURN
2260 IF A$="P2"  THEN LU=+1:LL=+1:JA=NA-1.5#:RETURN
2270 IF A$="Q2"  THEN LU=+1:LL=+1:JA=NA-.5# :RETURN
2280 IF A$="R2"  THEN LU=+1:LL=+1:JA=NA+.5# :RETURN
2290 IF A$="P12" THEN LU=-1:LL=+1:JA=NA-1.5#:RETURN
2300 IF A$="R21" THEN LU=+1:LL=-1:JA=NA+1.5#:RETURN
2310 PRINT " Bad Assignment !  Try again !! ":RETURN
2320 REM *********** A2H(3/2,1/2) UPPER LEVEL ************
2330 *GVU
2340 XX=0#:FOR IC=0 TO 4:XX=XX+RO^IC*OMGU(IC)*(V1+.5#)^IC:NEXT:GVU=XX:RETURN
2350 *FCU
2360 XX=0#:FOR IC=0 TO 2:XX=XX+RO^IC*A0(IC)*(V1+.5#)^IC:NEXT:A0=XX
2370 XX=0#:FOR IC=0 TO 7:XX=XX+RO^(IC+2)*BU(IC)*(V1+.5#)^IC:NEXT:BU=XX
2380 XX=0#:FOR IC=0 TO 3:XX=XX+RO^(IC+4)*DU(IC)*(V1+.5#)^IC:NEXT:DU=XX
2390 XX=0#:FOR IC=0 TO 1:XX=XX+RO^(IC+6)*HU(IC)*(V1+.5#)^IC:NEXT:HU=XX
2400 XX=0#:FOR IC=0 TO 2:XX=XX+RO^(IC+2)*P0(IC)*(V1+.5#)^IC :NEXT:P0=XX
2410 XX=0#:FOR IC=0 TO 2:XX=XX+RO^(IC+4)*Q0(IC)*(V1+.5#)^IC :NEXT:Q0=XX
2420 RETURN
2430 *FJU
2440 YV=A0/BU:X0=.5*SQR(YV*(YV-4)+4*(JA+.5)*(JA+.5))
2450 LAM1=(JA+.5)/X0*((1-.5*YV)*(.5*P0+Q0)+Q0*(JA-.5)*(JA+1.5))
2460 LAM2=(.5*P0+Q0)*(JA-.5)*(JA+1.5)/X0
2470 LAM3=.5*(JA+.5)*(.5*P0+Q0)
2480 IF A$="P1"  THEN PHAI=-LAM1-LAM2+LAM3:GOTO 2560
2490 IF A$="Q1"  THEN PHAI=+LAM1-LAM2-LAM3:GOTO 2560
2500 IF A$="R1"  THEN PHAI=-LAM1-LAM2+LAM3:GOTO 2560
2510 IF A$="P2"  THEN PHAI=-LAM1+LAM2-LAM3:GOTO 2560
2520 IF A$="Q2"  THEN PHAI=+LAM1+LAM2+LAM3:GOTO 2560
2530 IF A$="R2"  THEN PHAI=-LAM1+LAM2-LAM3:GOTO 2560
2540 IF A$="P12" THEN PHAI=+LAM1-LAM2-LAM3:GOTO 2560
2550 IF A$="R21" THEN PHAI=+LAM1+LAM2+LAM3:GOTO 2560
2560 REM UPPER ROTATION LEVEL
2570 IF LU=-1 THEN FJU=BU*((JA+.5)^2-1-X0)-DU*JA^4+HU*JA^6+PHAI:GOTO 2590
2580 IF LU=+1 THEN FJU=BU*((JA+.5)^2-1+X0)-DU*(JA+1)^4+HU*(JA+1)^6+PHAI
2590 RETURN
2600 *GEU
2610 A1=BU(1)*OMGU(1)/6/(BU(0)*BU(0))-1:A2=2*OMGU(2)/3/BU(0)+5*A1*A1/4
2620 GEU=BU*(2*3-1)+BU(0)/8*(3*A2-7*A1*A1/4)
2630 RETURN
2640 REM *********** X2Z(1/2) LOWER LEVEL ****************
```

```
2650 *GVL
2660 XX=0:FOR IC=0 TO 4:XX=XX+RO^IC*OMGL(IC)*(V2+.5)^IC:NEXT:GVL=XX
2670 *FCL
2680 XX=0:FOR IC=0 TO 3:XX=XX+RO^IC*GMA(IC)*(V2+.5)^IC:NEXT:GMA=XX
2690 XX=0:FOR IC=0 TO 5:XX=XX+RO^(IC+2)*BL(IC)*(V2+.5)^IC:NEXT:BL=XX
2700 XX=0:FOR IC=0 TO 3:XX=XX+RO^(IC+4)*DL(IC)*(V2+.5)^IC:NEXT:DL=XX
2710 XX=0:FOR IC=0 TO 1:XX=XX+RO^(IC+6)*HL(IC)*(V2+.5)^IC:NEXT:HL=XX
2720 RETURN
2730 *FJL
2740 FJL=BL*NA*(NA+1)-DL*(NA*(NA+1))^2+HL*(NA*(NA+1))^3
2750 IF LL=-1 THEN FJL=FJL+.5*GMA*NA     :GOTO 2770
2760 IF LL=+1 THEN FJL=FJL-.5*GMA*(NA+1)
2770 RETURN
2780 *GEL
2790 A1=BL(1)*OMGL(1)/6/(BL(0)*BL(0))-1:A2=2*OMGL(2)/3/BL(0)+5*A1*A1/4
2800 GEL=BL*(1*2-0)+BL(0)/8*(3*A2-7*A1*A1/4)
2810 RETURN
2820 *C2
2830 REM C2 LINE CALCULATION
2840 DIM VC2(10,1),AC$(10,10)
2850 DIM OMGUC(11),BUC(2,11),DUC(2,11),Y1UC(2,11),Y2UC(2,11)
2860 DIM OMGUD(11),BUD(2,11),DUD(2,11),Y1UD(2,11),Y2UD(2,11)
2870 DIM BLC(2,11),DLC(2,11),Y1LC(2,11),Y2LC(2,11)
2880 DIM BLD(2,11),DLD(2,11),Y1LD(2,11),Y2LD(2,11)
2890 FOR I=0 TO 0
2900 FOR J=0 TO 1:READ VC2(I,J):NEXT J
2910 NEXT I
2920 FOR I=0 TO 0
2930 FOR J=0 TO 1:READ AC$(I,J):NEXT J
2940 NEXT I
2950 DATA 1#,3#
2960 DATA P1,R1
2970 MU0C=6:MU1C=6.2407728#:MU2C=6.5016774#
2980 ROC(0)=1:ROC(1)=.98051996#:ROC(2)=.96064498#
2990 GOSUB *DATC
3000 FOR IAC=0 TO 0:V1C=VC2(IAC,0):V2C=VC2(IAC,1)
3010 FOR IAAC=0 TO 1:AC$=AC$(IAC,IAAC)
3020 FOR ILC=0 TO 2:ROC=ROC(ILC):GF=1#/89#^ILC
3030 FOR J1=0 TO 80
3040 GOSUB *ISO
3050 GLC=OMGLC*(V2C+.5)-OMGXL*(V2C+.5)^2
3060 GUC=OMGUC(V1C)*(V1C+.5)-OMGXU*(V1C+.5)^2+OMGYU*(V1C+.5)^3
3070 GOSUB *BRANCH
3080 GOSUB *ELECT
3090 YYC=NUEU-NUEL+GUC-GLC+FUC-FLC
3100 DNNC=.0002871#*(1+.00567#/(YYC*10000)^2)
3110 LAMDC=1/YYC-1/YYC*DNNC:LAMDC=LAMDC*100000000#
3120 IF LAMDC<LAMDAS OR LAMDC>LAMDAL GOTO 3140
3130 I3=I3+1:ABSL(I3,0)=LAMDC:ABSL(I3,1)=1#*GF
3140 NEXT J1,ILC,IAAC,IAC
3150 RETURN
3160 REM EVEN :KC=0 ODD KC=1  ex. BUC(KC,V1C) OR BLC(KC,V2C)
3170 *BRANCH
3180 IF AC$="P1" THEN GOTO 3250
3190 IF AC$="P2" THEN GOTO 3270
3200 IF AC$="P3" THEN GOTO 3290
3210 IF AC$="R1" THEN GOTO 3310
3220 IF AC$="R2" THEN GOTO 3330
3230 IF AC$="R3" THEN GOTO 3350
3240 RETURN
3250 JC=J1:GOSUB *LEO:GOSUB *FJ1:FLC=FJC:GOTO 3260
3260 JC=J1-1:GOSUB *UEO:GOSUB *FJ1:FUC=FJC:RETURN
3270 JC=J1:GOSUB *LEO:GOSUB *FJ2:FLC=FJC:GOTO 3280
3280 JC=J1-1:GOSUB *UEO:GOSUB *FJ2:FUC=FJC:RETURN
3290 JC=J1:GOSUB *LEO:GOSUB *FJ3:FLC=FJC:GOTO 3300
3300 JC=J1-1:GOSUB *UEO:GOSUB *FJ3:FUC=FJC:RETURN
3310 JC=J1:GOSUB *LEO:GOSUB *FJ1:FLC=FJC:GOTO 3320
3320 JC=J1+1:GOSUB *UEO:GOSUB *FJ1:FUC=FJC:RETURN
3330 JC=J1:GOSUB *LEO:GOSUB *FJ2:FLC=FJC:GOTO 3340
3340 JC=J1+1:GOSUB *UEO:GOSUB *FJ2:FUC=FJC:RETURN
3350 JC=J1:GOSUB *LEO:GOSUB *FJ3:FLC=FJC:GOTO 3360
3360 JC=J1+1:GOSUB *UEO:GOSUB *FJ3:FUC=FJC:RETURN
3370 *LEO
3380 IF (JC MOD 2)=1 THEN KC=1 ELSE KC=0
3390 BC=BLC(KC,V2C):DC=DLC(KC,V2C):Y1C=Y1LC(KC,V2C):Y2C=Y2LC(KC,V2C)
3400 RETURN
3410 *UEO
3420 IF (JC MOD 2)=1 THEN KC=1 ELSE KC=0
3430 BC=BUC(KC,V1C):DC=DUC(KC,V1C):Y1C=Y1UC(KC,V1C):Y2C=Y2UC(KC,V1C)
3440 RETURN
3450 *FJ3
3460 FJC=BC*(JC*(JC+1)+SQR(Y1C+4*JC*(JC+1))-(2/3#)*(Y2C-2*JC*(JC+1))/(Y1C+4*JC*(JC+1)))
3470 FJC=FJC+DC*(JC+1.5)^4
3480 RETURN
3490 *FJ2
3500 FJC=BC*(JC*(JC+1)-(4/3#)*(Y2C-2*JC*(JC+1))/(Y1C+4*JC*(JC+1)))+DC*(JC+.5)^4
3510 RETURN
3520 *FJ1
3530 FJC=BC*(JC*(JC+1)-SQR(Y1C+4*JC*(JC+1))-(2/3#)*(Y2C-2*JC*(JC+1))/(Y1C+4*JC*(JC+1)))
```

```
3540 FJC=FJC+DC*(JC-.5)^4
3550 RETURN
3560 *ELECT
3570 NUEU=19378.4#-71.55#:NUEL=0!
3580 AU=-16.9#:AL=-16.4#
3590 RETURN
3600 *ISO
3610 OMGLC=OMGLD*ROC:OMGXL=OMGXLD*ROC^2
3620 FOR IC=0 TO 10:OMGUC(IC)=ROC*OMGUD(IC):NEXT IC
3630 OMGXU=OMGXUD*ROC^2:OMGYU=OMGYUD*ROC^3
3640 FOR IC=0 TO 1:FOR KC=0 TO 10:BUC(IC,KC)=ROC^2*BUD(IC,KC):DUC(IC,KC)=ROC^4*DUD(IC,KC)
3650 Y1UC(IC,KC)=ROC^(-1)*Y1UD(IC,KC):Y2UC(IC,KC)=ROC^(-1)*Y2UD(IC,KC):NEXT KC:NEXT IC
3660 FOR IC=0 TO 1:FOR KC=0 TO 9 :BLC(IC,KC)=ROC^2*BLD(IC,KC):DLC(IC,KC)=ROC^4*DLD(IC,KC)
3670 Y1LC(IC,KC)=ROC^(-1)*Y1LD(IC,KC):Y2LC(IC,KC)=ROC^(-1)*Y2LD(IC,KC):NEXT KC:NEXT IC
3680 RETURN
3690 *DATC
3700 OMGLD=1641.35#:OMGXLD=11.67#
3710 FOR IC=0 TO 10:READ OMGUD(IC):NEXT IC
3720 OMGXUD=16.44#:OMGYUD=-.5067#
3730 FOR IC=0 TO 1
3740 FOR KC=0 TO 10:READ BUD(IC,KC),DUD(IC,KC),Y1UD(IC,KC),Y2UD(IC,KC):NEXT KC:NEXT IC
3750 FOR IC=0 TO 1
3760 FOR KC=0 TO 9:READ BLD(IC,KC),DLD(IC,KC),Y1LD(IC,KC),Y2LD(IC,KC):NEXT KC:NEXT IC
3770 DATA 1788.22#,1788.22#,1788.22#,1788.22#,1787.64#,1786.25
3780 DATA 1784.31#,0,1778.014#,1772.278#,1765.43#
3790 DATA 1.74557#,6.85D-6,127.29#,97.32#
3800 DATA 1.72535#,6.989D-6,127.29#,97.32#
3810 DATA 1.7043#,7.3D-6,127.29#,97.32#
3820 DATA 1.68145#,7.576D-6,127.29#,97.32#
3830 DATA 1.66119#,6.795D-6,127.29#,97.32#
3840 DATA 1.63865#,5.63D-6,127.29#,97.32#
3850 DATA 1.61035#,5.63D-6,127.29#,97.32#
3860 DATA 0,0,0,0
3870 DATA 1.5267#,11.4D-6,127.016#,97.12#
3880 DATA 1.4845#,11.1D-6,95.31#,76.77#
3890 DATA 1.44156#,11.03D-6,127.29#,97.32#
3900 DATA 1.74544#,6.856D-6,127.29#,97.32#
3910 DATA 1.72511#,6.992D-6,127.29#,97.32#
3920 DATA 1.7043#,7.3D-6,127.29#,97.32#
3930 DATA 1.68145#,7.576D-6,127.29#,97.32#
3940 DATA 1.6609#,6.595D-6,127.29#,97.32#
3950 DATA 1.63865#,5.33D-6,127.29#,97.32#
3960 DATA 1.61035#,5.33D-6,127.29#,97.32#
3970 DATA 0,0,0,0
3980 DATA 1.5267#,11.4D-6,122.32#,93.01#
3990 DATA 1.4845#,11.1D-6,97.71#,58.77#
4000 DATA 1.44156#,12.53D-6,127.29#,97.32#
4010 DATA 1.6237#,6.4D-6,158.6981#,124.7987#
4020 DATA 1.60715#,6.42D-6,161.2349#,127.0113#
4030 DATA 1.59043#,6.43D-6,163.6433#,129.1189#
4040 DATA 1.57362#,6.43D-6,166.0543#,131.1238#
4050 DATA 1.557#,6.54D-6,168.2237#,133.1099#
4060 DATA 1.54038#,6.54D-6,170.7597#,135.3192#
4070 DATA 1.52388#,7.05D-6,173.1433#,137.405#
4080 DATA 1.50724#,7.05D-6,175.5733#,139.7518#
4090 DATA 1.4906#,7.05D-6,177.9183#,141.8214#
4100 DATA 1.47396#,5.5D-6,180.34333,143.9784#
4110 DATA 1.6237#,6.4D-6,156.8223#,123.1407#
4120 DATA 1.60715#,6.42D-6,159.4936#,125.412#
4130 DATA 1.59043#,6.43D-6,161.6933#,127.4094#
4140 DATA 1.57362#,6.43D-6,164.1714#,129.5753#
4150 DATA 1.557#,6.54D-6,166.3201#,131.4527#
4160 DATA 1.5402#,6.75D-6,168.517#,133.3746#
4170 DATA 1.52362#,7.26D-6,171.0838#,135.6214#
4180 DATA 1.50694#,7.26D-6,173.4183#,137.815#
4190 DATA 1.49026#,7.26D-6,175.8033#,139.9415#
4200 DATA 1.47358#,5.5D-6,178.0933#,141.9647#
4210 RETURN
4220 *SORT
4230 REM QUICK SORTING
4240 IPS=1:ILS=1:IUS=13:IEMPTY=0
4250 IS=ILS:JS=IUS:RKEY=ABSL((IS+JS)¥2,0)
4260 WHILE RKEY>ABSL(IS,0)
4270 IS=IS+1
4280 WEND
4290 WHILE RKEY<ABSL(JS,0)
4300 JS=JS-1
4310 WEND
4320 IF IS<=JS THEN SWAP ABSL(IS,0),ABSL(JS,0):SWAP ABSL(IS,1),ABSL(JS,1):IS=IS+1:JS=JS-1:IF IS<=JS THEN 4260
4330 IF (IUS-IS)<(JS-ILS) THEN 4370
4340 IF IUS>IS THEN IAS=IS:IBS=IUS:GOSUB *PUSH
4350 IUS=JS
4360 GOTO 4390
4370 IF JS>ILS THEN AS=ILS:IBS=JS:GOSUB *PUSH
4380 ILS=IS
4390 IF IUS>ILS THEN 4250
4400 GOSUB *POP
4410 IF NOT IEMPTY THEN 4250
4420 RETURN
```

```
4430 *PUSH
4440 IF IPS>13 THEN PRINT "STack Overflow":RETURN 4420
4450 ITS(IPS,1)=IAS:ITS(IPS,2)=IBS:IPS=IPS+1
4460 RETURN
4470 *POP
4480 IPS=IPS-1
4490 IF IPS=0 THEN IEMPTY=-1:RETURN
4500 ILS=ITS(IPS,1):IUS=ITS(IPS,2)
4510 RETURN
```