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Synthetic Spectra Method for Carbon Stars

II. Some calculations from the realistic data

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炭素星に対するスペクトル合成法

Ⅱ．現実的なデータによる計算

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要　旨

スペクトル合成法は，理論的に計算された合成スペクトルと観測されたスペクトルを比較して，恒星大気の物理量や化学組成を求める方法で，とくに炭素星のように連続スペクトルのレベルの決定が困難な恒星の解析に有効である．この方法は大量の計算と記憶容量を要するため，従来は大型計算機を用いてなされている．また，パーソナル・コンピューターを用いる場合には，ミンナートの近似式で合成スペクトルが計算されるが，この場合，大気の複雑な構造を考慮に入れることができない．

炭素星のD線付近のスペクトルの解析を目ざして，本研究年報の第7号で，筆者はミンナートの近似式によらないより精度の高い合成スペクトルをパーソナル・コンピューターで計算する方法を試みた．その際，筆者らがすでに開発している放射流束のガウス型数値積分の式により合成スペクトルを計算するプログラムを開発した．しかし，その計算では計算時間短縮のため，分子の線のgf値や線輪郭について非現実的な仮定がなされた．

本研究で筆者は，それらより現実的な仮定のもとに合成スペクトルを計算するプログラムを開発した．すなわち，gf値として振動および回転振動子強度を考慮し，さらにボルツマン因子を考慮した．また，Voigt型輪郭を線輪郭とした．そして，辻隆氏の計算した炭素星のモデル大気（T_{eff}=3400K，g=0.1cgs）をもとに，5885～5900Åの波長域のCN分子による合成スペクトルが，本プログラムで計算された．PC－9801VM2を用いての計算時間は14.5分であった．本プログラムでは，まだ吸収線にかかわるエネルギー準位にある数密度の大気内での変化が考慮されていない．しかし，それを考慮しても，高精度の合成スペクトルが実用的な時間内で計算されるであろうことを，上述の計算時間は示唆している．

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I. Introduction

Carbon stars are the stars with late spectral type whose spectra are characterized by the predominance of the bands of carbon compounds such as CH, CN and C_2. Dissociative-equilibrium calculations have shown that the spectral characteristics of carbon stars result from the peculiar chemical compositions of the atmospheres. Carbon is more abundant than oxygen in the atmospheres of carbon stars, while oxygen is more abundant than carbon in the atmospheres with normal chemical compositions like the sun. It is widely believed that the peculiar chemical compositions of the atmospheres of carbon stars result from the mixing of the atmospheres with the inner layers which have undergone nuclear processing, but details of the mixing are not yet elucidated.

Carbon stars are classified into R and N types in the Harvard classification system, which was improved by Shane (1928)[1] who introduced subtypes. It has since become clear that the color temperatures used in the Harvard R -N system do not correspond to the surface temperature of the atmospheres. Keenan and Morgan (1941)[2] have proposed a new classification system of carbon stars which is known as the C-classification system. This system was improved by Yamashita (1967)[3] and many carbon stars were classified on this system by him. This system is two dimensional and two parameters are used to classify carbon stars. One parameter is expected to indicate the surface temperature and is expressed on a scale from 0 to 9 (lowest temperature). The other parameter is expected to indicate the carbon abundance and is expressed on a scale from 1 to 5 (most abundant). For example, the carbon star Y CVn is classified on this system as C5, 4 where the first figure following the letter C gives the temperature class and the next figure gives the carbon-abundance class. The carbon-abundance class is often omitted. In this case, the C-classification type of Y CVn is expressed as C5.

Although C-classification system is now most commonly adopted, some doubt about the validity of this system as a temperature sequence has been expressed by several investigators. For example, Tsuji (1981)[4] determined the effective temperatures of thirty-one carbon stars by the infrared flux method and he found these effective temperatures show poor correlation, or even anti-correlation, with the temperature classes of the C-classification. He concluded that this poor correlation is attributed to the poor correlation of the effective temperatures with the strengths of sodium D lines which are the major temperature class criterion of C-classification. He suggested that the blanketing effect by CN bands may cause the insensitivity of the strengths of D lines to the effective temperatures. He also suggested that the blend effect by interstellar or circumstellar D lines and by the strong CN bands near the D line may lead to the inaccuracy in the determination of the temperature class of C-classification.

Formerly, coarse analyses using curve-of-growth have exclusively done for carbon stars. Recently, fine analyses also have been done for these stars. For carbon stars,

synthetic spectra methods rather than curve-of-growth methods have been utilized in fine analyses, because it is difficult to measure accurate values of equivalent widths owing to the uncertainty of continuum. The synthetic spectra methods in fine analyses require a large amount of memory capacity and have been applied only by a large-sized computer. The synthetic spectra methods have been utilized in coarse analyses, too, and have been applied also by a personal-computer[5].

Yoshioka (1989)[6] (hereafter referred to as Paper I) developed a personal-computer program for synthetic spectra methods in fine analyses in order to check the validity of the suggestion by Tsuji (1981)[4] as to D lines of carbon stars. The divided 4-points Gaussian quadrature formula developed by Nariai and Yoshioka (1983)[7] was modified and used for the calculation of flux. In order to test his program, Yoshioka (1989)[6] calculated synthetic spectra in the wavelength range from 5885Å to 5900Å where D lines exist, but these spectra were obtained from the data on lines which are not realistic at all. In this paper, synthetic spectra of carbon stars in these wavelength range have been calculated from more realistic data on lines.

II. The Principle in the Present Program

II.1. The Principle of Synthetic Spectra Methods

In synthetic spectra methods, the synthetic spectra of a program star are calculated assuming physical quantities and chemical composition of the atmosphere, and these spectra are compared with the observed spectra of the program star. In case that both of the spectra agree well, the assumed physical quantities and chemical composition are adopted as thoses for the program star. Otherwise, spectra are synthesized with other physical quantities and chemical composition, and the process is repeated until synthesized spectra agree well with the observed ones.

In coarse analyses, the Minnaert Formula (Minnaert (1935)[8]) is usually used for a spectral synthesis calculation, which is

$$\frac{1}{R_\lambda} = \frac{1}{R_c} + \frac{1}{x_\lambda}, \tag{1}$$

where R_λ is the depth of a line at a wavelength λ, R_c is the limiting central depth of the line, and x_λ is the effective relative optical depth at λ. The effective relative optical depth is the optical depth of a line divided by the optical depth of the continuum near the line which is multiplied by a constant so as to agree with R_λ for weak lines. Using the astrophysical flux F_λ (energy flux divided by π) at a stellar surface, R_λ is defined by

$$R_\lambda = 1 - \frac{F_\lambda}{F_c}, \tag{2}$$

where F_λ is the flux of the line at λ and F_c is the flux of the continuum near the line. The Minnaert formula holds true for only the Milne-Eddington atmospheric model with a source function depending linearly on the optical depth.

In fine analyses, both F_λ and F_c are calculated from a model atmosphere, the changes of the physical quantities in the atmosphere being taken into account, and R_λ is obtained through equation (2). In many cases, F is calculated using the following integral:

$$F=2\int_0^\infty S(\tau)\,E_2(\tau)\,d\tau\,, \tag{3}$$

where $S(\tau)$ is the source function at the optical depth τ and $E_2(\tau)$ is the exponential integral function of order 2. The exponential integral function of order n is defined by

$$E_n(\tau)=\int_1^\infty \frac{\exp(-\tau t)}{t^n}\,dt\,. \tag{4}$$

II.2. The Calculation in Paper I

In Paper I, the integral (3) was calculated on basis of the divided 4-points Gaussian quadrature formula which was developed by Nariai and Yoshioka (1983)[7] and modified in Papar I. According to this formula, the integral (3) is divided into six subranges of (0, 0.01), (0.01, 0.1), (0.1, 0.3), (0.3, 1.0), (1.0, 12.0) and (12.0, ∞), and except for the last subrange (12.0, ∞) the integral for each subrange is calculated by the following 4-points Gaussian quadrature formula for the weight function $E_2(t)$:

$$\int_x^y S(t)\,E_2(t)\,dt=\sum_{i=1}^{4} a_i S(t_i)\,, \tag{5}$$

where a_i's are weights and t_i's are corresponding weighting points. The values of a_i and t_i are given in table 1 in Paper I. The integral for the subrange (12.0, ∞) is neglected, because the value of the integral for this subrange is very small compared with the value of the integral for the entire range. For example, the ratio is equal to 0.00085% in case of the gray atmosphere. The relative error, $\Delta F/F$, for the gray atmosphere calculated by this formula is equal to -0.00043%.

In Paper I, the synthetic spectrum was calculated in the following way. First, for each wavelength, both F_λ and F_c were calculated according to the formula (5). Then, R_λ was calculated according to equation (2) and the synthetic spectrum, i. e., the relationship between R_λ and λ was obtained. In the calculation according to the formula (4), it was assumed that the atmosphere is in local thermodynamic equilibrium and the line absorption occurs according to the mechanism of pure absorption. The following relation holds for the source function $S_\lambda(\tau_\lambda)$ at λ:

$$S_\lambda(\tau_\lambda)=B_\lambda(T)\,, \tag{6}$$

where $B_\lambda(T)$ is the Planck function at λ for the temperature T corresponding to the optical depth τ_λ. Thus, in Paper I, both F_λ and F_c were calculated according to the following formula:

$$F=\sum_{j=1}^{5}\{\sum_{i=1}^{4}a_iB_\lambda(T_i)\}, \qquad (7)$$

where T_i is the temperature corresponding to the weigting point t_i. The value of T is obtained via the relationship between $\log_{10}B_{\rm std}(T)$ and $\log_{10}\tau_{\rm std}$, where $B_{\rm std}(T)$ is the Planck function at the standard wavelength and $\tau_{\rm std}$ is the optical depth at the standard wavelength. This relationship is used, because $\log_{10}B_{\rm std}(T)$ varies approximately linearly with $\log_{10}\tau_{\rm std}$. The calculation of the optical depth from the absorption coefficient is made by the use of divided 6-points Gauss-Legendre's formula.

In Paper I, the program for the synthetic spectra method was tested with one of the model atmospheres for carbon stars calculated by Querci et al. (1974)[9]. These are the first model atmospheres for carbon stars where the molecular line blanketing effect by CO, CN, C_2 is taken into account through opacity probability distribution functions. In these model atmospheres, it is assumed that the following hypotheses are valid: radiative equilibrium, local thermodynamic equilibrium, plane-parallel layers, and steady state. The microturbulence velocity is assumed to be equal to 5 km/s throughout the atmosphere. The relative H/He/C/N/O abundance adopted in the computation are as follows:

$$\mathrm{He/H}=0.16,\ \ \mathrm{C/H}=4.1\times10^{-5},\ \ \mathrm{N/H}=1.48\times10^{-3},\ \ \mathrm{O/H}=1.25\times10^{-5}.$$

The model used in Paper I is the one for the effective temperature $T_{\rm eff}=3400$ and the surface gravity $g=0.1$ cgs. It gives the physical quantities as a function of the optical depth at the standard wavelength ($=0.88\,\mu$). The same model also used in the present paper. The details of this model are given in table 2 in Paper I.

In Paper I, the following assumptions were made concerning to the absorption coefficients of molecular lines.

a) All the $^{12}C^{14}N$ and $^{12}C^{12}C$ lines have the same gf value. All the $^{13}C^{14}N$ lines have the same gf value which is smaller than that for $^{12}C^{14}N$ lines by a factor of 89 (=the solar $^{12}C/^{13}C$ ratio). All the $^{13}C^{13}C$ lines have the same gf value which is smaller than that for $^{12}C^{12}C$ lines by a factor of 7921 ($=89\times89$).

b) The ratio of the line absorption coefficient to the continuous absorption coefficient is constant with optical depth. Furthermore, the continuous absorption coefficient is independent of wavelength.

c) The shape of the line absorption coefficient is a isosceles triangle where the vertex corresponds to the line center. The altitude of the triangle is in proportion to the gf value of the line.

d) All the lines have the same width.

Figure 1 shows the synthetic spectrum calculated with the program based on the above assumptions which is listed in Paper I. The wavelength ranges from 5885Å to 5900Å. The step in the calculation is 0. 1Å. Only the P_1 and R_1 branches of the (11, 5) band of the CN red system (9 lines) and the P_1 and R_1 branches of (1, 3) band of the C_2 Swan system (14 lines) are taken into account. (There is some misprints in Paper I in the description of the lines which are taken into account in the calculation.) The ratio of the line absorption coefficient at line center for $^{12}C^{14}N$ and $^{12}C^{12}C$ to the continuous absorption coefficient is taken to be 25. The full widths of all the lines are taken to be 1.0Å.

II.3. The Calculation in the Present Program

In the present program, the assumptions in Paper I concerning to the absorption coefficients of molecular lines are replaced by more realistic ones, except for the assumption b).

As for the assumption c), the Voigt profile is taken as the shape of the absorption coefficient. The Voigt profile is the convolution of the Lorentzian and Gaussian profiles and it is the most realistic line profile. It is defined by the following convolution integral:

$$H(a, v) \equiv \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{\exp(-y^2)}{(v-y)^2+a^2}\, dy. \tag{8}$$

In the above integral, v and a stand for the following quantities:

$$v \equiv \frac{\lambda-\lambda_0}{\Delta\lambda_D}, \tag{9}$$

and

$$a \equiv \frac{\Gamma}{4\pi c}\,\frac{\lambda_0^2}{\Delta\lambda_D}, \tag{10}$$

where c is the speed of light and λ_0 is the wavelength at the center of a line; Γ is the effective damping constant and $\Delta\lambda_D$ is the Doppler width in wavelength units. Here, $\Delta\lambda_D$ includes contributions of thermal motion and turbulence and it is given as follows:

$$\Delta\lambda_D = \frac{\lambda_0}{c}\sqrt{\frac{2kT}{M}+\xi^2}, \tag{11}$$

where k is Boltzmann's constant and ξ is the turbulent velocity; M is the mass of molecule.

The values of $H(a, v)$ are usually calculated by an expansion of the following form:

$$H(a, v) = \sum_{n=0}^{4} a_n H_n(v), \tag{12}$$

where the functions $H_n(v)$ are tabulated by Harris (1948)[10] for $v \leqq 12$. However, the error in the formula (12) increases with the increase of the value of a, and for $a \geqq 0.3$ the

absolute error exceeds 10^{-4} whose value is set up as the lower limit of the ratio of the absorption coefficient of the weakest line to the one of the strongest line.

Thus, in the present program, the following asymptotic formulae are used for the calculation of $H(a, v)$ according to Reichel (1968)[11]:

$$H(a, v)=\frac{\exp(-v^2)}{a\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{v^{2n}}{n!}U_n, \tag{13}$$

and

$$H(a, v)=\frac{1}{v\sqrt{\pi}}\sum_{n=0}^{\infty}(-1)^n W_{2n-1}, \tag{14}$$

where U_n's and W_n's satisfy the following asymptotic relations:

$$U_n=\frac{2a^2}{2n-1}(1-U_{n-1}), \tag{15}$$

and

$$W_n=\frac{a}{v}W_{n-1}-\frac{n-1}{2v^2}W_{n-2}. \tag{16}$$

In theses relations, the values of W_0, W_1, and U_0 are calculated by the following formulae:

$$W_0=-1, \tag{17}$$

$$W_1=-a/v, \tag{18}$$

and

$$U_0=\sum_{n=0}^{\infty}u_n, \tag{19}$$

where u_n's satisfy the following asymptotic relations:

$$u_0=a\sqrt{\pi}, \tag{20}$$

$$u_1=-2a^2, \tag{21}$$

and

$$u_n=\frac{2a^2}{n}u_{n-2}. \tag{22}$$

For smaller values of v the formula (13) is available, while for larger values of v the formula (14) is available. In the present program, the formula (13) is used for $v\leqq 4.2$, and the formula (14) is used for $v>4.2$. By the use of these formula the absolute error can be held within 10^{-4} for all the values of a and v. Moreover, except for $\sim 1.5<v\leqq 4.2$, the time of the calculation by these formulae is smaller than that by the formula (12). (For the calculation by the formula (12) the value of $H_n(v)$ needs to be interpolated with a high accuracy from the table by Harris (1948)[10], which takes time.)

As for the assumption a), the gf value of a molecular line is assumed to be proportional to the rotational line strength multiplied by the oscillator strength for the vibrational transition. Moreover, the Boltzmann factor is taken into account in the calculation of the line absorption coefficient. Thus, the following proportionality are assumed for the line absorption coefficient per gram at the line center κ_0:

$$\kappa_0\propto f_r S_J\exp(-\chi_1/kT_{ex}), \tag{23}$$

where f_v is the oscillator strength for the vibrational transition and S_J is the rotational line strength; χ_1 is the excitation potential of the lower energy level and T_{ex} is the excitation temperature.

As for the assumption d), the width of the line is determined as a natural consequence of the above mentioned modifications of the assumptions a) and c).

Moreover, in the present program, the calculating time of line absorption coefficient at a given wavelength is shortened by avoiding a survey of all the lines. The avoidance is done in the following way. In the first place, all the lines are arranged by the shorter end of line starting with the smallest one. Then, for a given wavelength the lines having nonzero absorption coefficient are surveyed according to the algorithm which is shown in figure 3. This avoidance shortens the calculating time by more than a factor of about 5.

III. Results and Discussion

The present program is written in BASIC and is named "FLUX 1". It is listed in the Appendix. A personal-computer PC-9801VM2 (NEC) with double precision floating number is used throughout the calculation. In the actual calculation, the program is compiled and is run with V 33 super charger in order to shorten the calculating time.

For the sake of exhibition of the effect of the modification of the assumption c), figure 2 shows the synthetic spectrum calculated with the present program, where the other assumptions are the same as in Paper I. Both the wavelength range and the step in the calculation are the same as in figure 1. The lines taken into account are the same as in figure 1, and the ratio of the line absorption coefficient at line center for $^{12}C^{14}N$ and $^{12}C^{12}C$ to the continuous absorption coefficient also is the same as in figure 1. For all the lines the Doppler width is taken to be 1/3Å, and the value of a is taken to be 0.01. It takes 12 minutes and 48 seconds to calculate the spectrum in figure 2, while it takes 5 minutes and 28 seconds to calculate the spectrum in figure 1. Figure 2 indicate that, except for the effect caused by the difference in the line width, the profiles for the lines with small width does not differ remarkably from those in figure l.

Figure 4 shows the synthetic spectrum calculated with the present program where all the modifications are done. Both the wavelength range and the step in the calculation are the same as in figure 1. Only $^{12}C^{14}N$ lines are taken into account. On the basis of the data offered by Hirai (1991)[12] 85 lines are included. In the calculation of the Doppler width by the formula (11), T is taken to be 4039K and ξ is taken to be 5 km/s. The ratio of the line absorption coefficient at line center of the strongest line to the continuous absorption coefficient is taken to be 10. The line whose ratio of the line absorption coefficient to the continuous absorption coefficient is smaller than 0.01 are not taken into account. It takes 14 minutes and 29 seconds to calculate this spectrum.

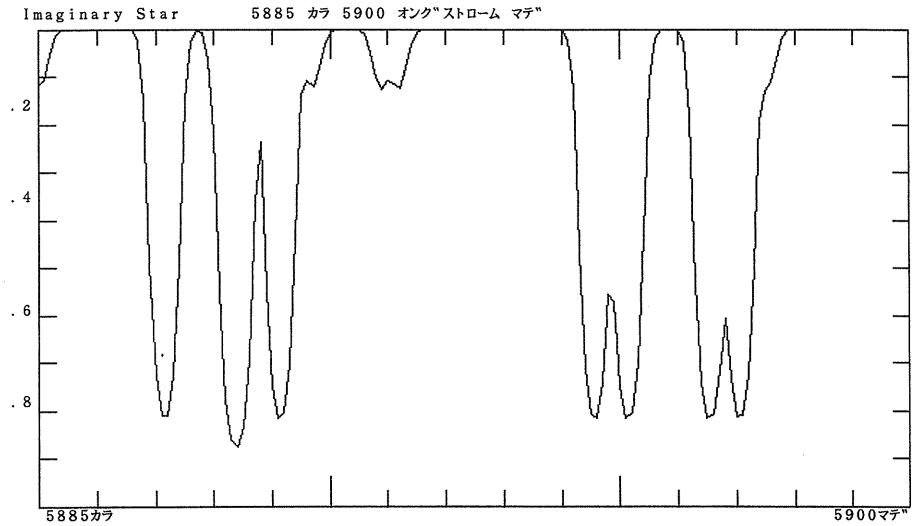

Fig. 1. The synthetic spectrum calculated with the program in Paper I. The wavelength ranges from 5885Å to 5900Å. Only the 9 lines of the CN red system and 14 lines of the C_2 Swan system are taken into account. The full width of the line is taken to be 1.0Å for all the lines.

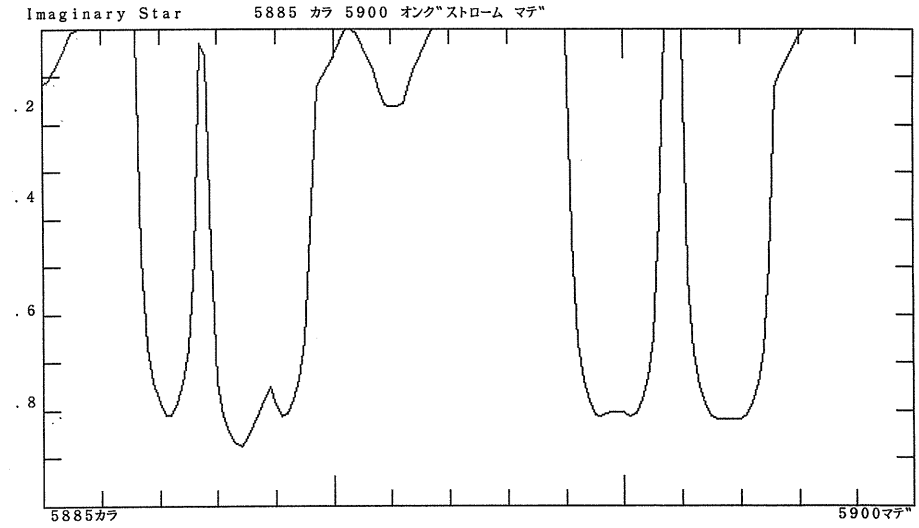

Fig. 2. The synthetic spectrum calculated with the present program. Both the wavelength range and the lines considered are the same as in figure 1. The Doppler width is taken to be 1/3Å for all the lines.

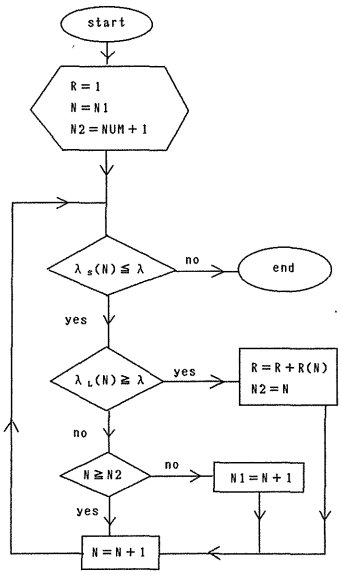

Fig. 3. The flow chart of the algorithm which is used in the present program for the survey of lines in the calculation of line absorption coefficient. Before the start all the lines are arranged by the shorter end of line starting with the smallest one. In this chart, NUM is the total number of lines and N is the ordinal number of the line under survey; λ is the wavelength at which the absorption coefficient is calculated; $\lambda_S(N)$ and $\lambda_L(N)$ are the wavelength of the shorter and longer end of the line under survey, respectively; R(N) is the ratio of the line absorption coefficient to the continuous absorption coefficient for the line under survey and R is the sum of R(N)'s.

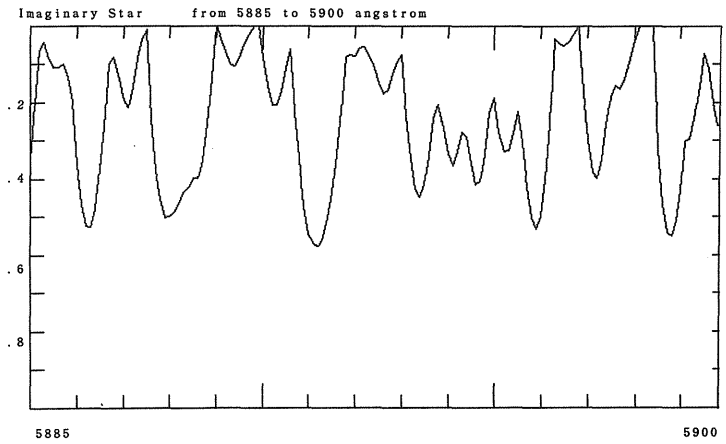

Fig. 4. The synthetic spectrum calculated with the present program. The wavelength range is the same as in figure 1. Here, 85 lines of $^{12}C^{14}N$ are taken into account. For each line the Doppler width is calculated by the formula (11), where T is taken to be 4039K and ξ is taken to be 5 km/s.

The present results are obtained on the basis of more realistic assumptions than those in Paper I, but the assumption b) remains to be replaced by more realistic one. Although the consideration of this lengthens the calculating time, the calculating time of the spectrum in figure 4 (=14 minutes and 29 seconds) is short enough to suggest the prospects of practical use of a personal computer for realistic synthetic spectra. On the basis of the present results, the improvements of the present program are being made by the author.

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Appendix. List of the Program "FLUX1"

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10 REM Program "FLUX1"
20 REM Calculation of Relative Flux at the Surface by the Quadrature Formula of Nariai and Yoshioka
30 REM Calculation of Source Function from the Table of T vs log(Tau((standard))
40 REM T Value for Given log(Tau) Is Interpolated Via Planck Function at 0.88 Micron
50 REM Tau(at 0.88 micron) corresponding to Tau(line) is calculated by Gauss's quadrature formula
60 REM by QQT Model:Te=3400K,κ=0.1cgs
70 REM Data on Molecules are Input by Tabular Form
80 DEFDBL A-H,L-Z:DEFINT I-K
90 DIM AW(20),DP(20),S(20),FL(2)
100 DIM LGTAU(41),T(41),TA(41),DP1(3),DP2(6),BW(3),DPL(30),TAUL(30),DP3(20),LR(20),AR(6)
110 DIM ABSL(300,4),ITS(13,2)
120 DIM U(100),SS(300),VV(300):RP=SQR(3.141592653589793#)
130 C=.5772156649015329#:E1=4.751080258825322D-07:LN=LOG(10#)
140 FOR I=1 TO 20:READ AW(I):NEXT I
150 FOR I=1 TO 20:READ DP(I):NEXT I
160 DATA 0.00172099#,0.00318973#,0.00314859#,0.00166413#,0.014111#,0.024890#,0.023209#,0.011774#
170 DATA 0.023299104#,0.039878941#,0.035610382#,0.017461205#,0.046986375#,0.069873539#,0.051680974#,0.021808971#
180 DATA 0.067878833#,0.037638504#,0.004094547#,0.000079668#
190 DATA 0.00069017#,0.00328735#,0.00668802#,0.00930234#,0.010061#,0.039093#,0.069725#,0.093589#
200 DATA 0.113252253#,0.163873065#,0.231908553#,0.285510191#,0.342970767#,0.511559098#,0.749107458#,0.945450967#
210 DATA 1.252874435#,2.407230937#,4.740536006#,8.667433737#
220 LGTAU(0)=-8#:LGTAU(1)=-7#
230 FOR I=2 TO 41:LGTAU(I)=-6#+.2#*(I-2):NEXT I
240 T(0)=1877#:T(1)=1889#
250 FOR I=2 TO 41:READ T(I):NEXT I
260 DP(0)=0#:DPL(0)=0#
270 FOR I=1 TO 3:READ DP1(I):NEXT I
280 FOR I=1 TO 3:READ BW(I):NEXT I
290 DATA 1920#,1934#,1942#,1956#,1974#,1999#,2030#,2070#,2113#,2158#
300 DATA 2203#,2252#,2303#,2357#,2415#,2476#,2540#,2610#,2689#,2777#
310 DATA 2872#,2969#,3071#,3174#,3279#,3388#,3502#,3624#,3750#,3887#
320 DATA 4039#,4208#,4424#,4718#,5103#,5579#,6144#,6761#,7397#,8073#
330 DATA 0.9324695142031152#,0.6612093864662645#,0.2386191860831969#
340 DATA 0.1713244923791703#,0.3607615730481386#,0.467913934572691#
350 FOR I=0 TO 41:TA(I)=16349.64#/T(I):TA(I)=-LOG(EXP(TA(I))-1#):NEXT I
360 INUM0=95:GFMAX=0:AH=.01#:LGAH=LOG(AH)/LOG(10#)
370 TEMP=4039#:VMICRO=5#:ATW=26.003#:RMAX=10#:RESTORE 3000
380 FOR I=1 TO INUM0
390 READ ABSL(I,0),ABSL(I,3),EXCP
400 VH=0:GOSUB *VOIGT:ABSL(I,4)=HAV:ABSL(I,3)=ABSL(I,3)*HAV*EXP(-11604#*EXCP/TEMP)
410 IF ABSL(I,3)>GFMAX THEN GFMAX=ABSL(I,3)
420 NEXT I
430 RFAC=RMAX/GFMAX
440 INPUT "Wavelength(Shortest,Longest,Step,Mark)";LAMDAS,LAMDAL,DLAMDA,MLAMDA
450 GOSUB *LINEWIDTH:ABSL(INUM+1,1)=100000#
460 GOSUB *SORT
470 TIME$="00:00:00"
480 LAMDA=LAMDAS:I2=1:I4=1
490 I1=CINT(LAMDAL-LAMDAS)/DLAMDA+1
500 DIM LENG(I1),FLU(I1)
510 WHILE LAMDA<=LAMDAL+.1#*DLAMDA
520 GOSUB *CALCULATION
530 LENG(I2)=LAMDA:FLU(I2)=XF
540 LAMDA=LAMDAS+I2*DLAMDA:I2=I2+1
550 WEND
560 LPRINT "Calculation Time = ";TIME$:LPRINT "Max. Raitio = ";RMAX;"    log(Alpha) = ";LGAH:LPRINT
570 BEEP
580 INPUT "Do You Want to Show the Result at the Display ? Y or N";ANS$
590 IF ANS$="Y" THEN GOSUB *GRAPH1
600 END
610 *CALCULATION
620 REM Calculation of Relative Flux
630 FOR I0=1 TO 2
640 GOSUB *SOURCE
650 XF=0#
660 FOR I=1 TO 20:XF=XF+AW(I)*S(I):NEXT I
670 FL(I0)=1.5#*XF
680 NEXT I0
690 XF=FL(2)/FL(1)
700 RETURN
710 *GRAPH1
720 REM Display of Relative Flux vs Wavelength
730 CONSOLE 0,25,0,0:CLS 3:SCREEN 2,0,0,1:COLOR 0
740 DA=(LAMDAL-LAMDAS)/50
750 WINDOW(LAMDAS-DA*2.3,-.05)-(LAMDAL,1.1):VIEW(0,0)-(639,385)
760 INPUT "Name of the Star";NAME$
770 A$="from":AA$="to"
780 LOCATE 2,2
790 PRINT NAME$;SPC(5);A$;LAMDAS;AA$;LAMDAL;"angstrom"
800 FOR I=1 TO 4
810 LOCATE 0,(1-1)*4.3+6:PRINT .2*I
820 NEXT I
830 B$=STR$(LAMDAS):C$=STR$(LAMDAL)
840 LOCATE 0,25:PRINT SPC(3);B$;SPC(66);C$
```

```
 50 LINE(LAMDAS,1)-(LAMDAL,0),,B
 60 FOR I=1 TO 9
 70 LINE(LAMDAS,I/10)-(LAMDAS+DA,I/10):LINE(LAMDAL,I/10)-(LAMDAL-DA,I/10)
 80 NEXT I
 90 DB=LAMDAS+1
100 WHILE DB<LAMDAL
110 IF DB MOD MLAMDA=0 GOTO 930
120 LINE(DB,1)-(DB,.97):LINE(DB,0)-(DB,.03):GOTO 940
130 LINE(DB,1)-(DB,.94):LINE(DB,0)-(DB,.06)
140 DB=DB+1
150 WEND
160 FOR I=1 TO I2-2
170 LINE(LENG(I),1#-FLU(I))-(LENG(I+1),1#-FLU(I+1))
180 NEXT I
190 ANS$=INKEY$
200 IF ANS$="" GOTO 990
210 CLS 3:LOCATE 0,0,1:CONSOLE ,,1
220 RETURN
230 *SOURCE
240 REM Claculation of Source Function
250 TAU=.1#:GOSUB *ABS.RATIO
260 FOR I=1 TO 20:DPL(I)=DP(I)/RR:NEXT I
270 TAUL(0)=0#:TAU=DPL(1):GOSUB *ABS.RATIO:TAUL(1)=RR*DPL(1):I=1
280 WHILE TAUL(I)<DP(20)
290 IF I=20 GOTO 1170
300 I=I+1:A=(DPL(I)-DPL(I-1))/2#:B=(DPL(I)+DPL(I-1))/2#:TAUL(I)=TAUL(I-1)
310 FOR J=1 TO 3:DP2(J)=-A*DP1(J)+B:DP2(7-J)=A*DP1(J)+B:NEXT J
320 FOR J=1 TO 6:TAU=DP2(J):GOSUB *ABS.RATIO:AR(J)=A*RR:NEXT J
330 FOR J=1 TO 3:TAUL(I)=TAUL(I)+AR(J)*BW(J):NEXT J
340 FOR J=4 TO 6:TAUL(I)=TAUL(I)+AR(J)*BW(7-J):NEXT J
350 WEND
360 GOTO 1250
370 TAU=DPL(20):GOSUB *ABS.RATIO:DTAU=(DP(20)-DP(19))/RR:A=DTAU/2#
380 WHILE TAUL(I)<DP(20)
390 I=I+1:B=DPL(20)+DTAU*(I-20.5#):DPL(I)=DPL(I-1)+DTAU:TAUL(I)=TAUL(I-1)
400 FOR J=1 TO 3:DP2(J)=-A*DP1(J)+B:DP2(7-J)=A*DP1(J)+B:NEXT J
410 FOR J=1 TO 6:TAU=DP2(J):GOSUB *ABS.RATIO:AR(J)=A*RR:NEXT J
420 FOR J=1 TO 3:TAUL(I)=TAUL(I)+AR(J)*BW(J):NEXT J
430 FOR J=4 TO 6:TAUL(I)=TAUL(I)+AR(J)*BW(7-J):NEXT J
440 WEND
450 II=I:JJJ=1
460 FOR I=1 TO 20
470 DP3(I)=0#
480 FOR J=JJJ TO II
490 IF DP(I)<TAUL(J) GOTO 1310
500 NEXT J
510 JJJ=J
520 IF J=1 GOTO 1420
530 IF J=II GOTO 1500
540 FOR K=-2 TO 1
550 AA=1#:BB=1#
560 FOR JJ=-2 TO 1
570 IF JJ<>K THEN AA=AA*(DP(I)-TAUL(J+JJ)):BB=BB*(TAUL(J+K)-TAUL(J+JJ))
580 NEXT JJ
590 DP3(I)=DP3(I)+DPL(J+K)*AA/BB
600 NEXT K
610 GOTO 1570
620 FOR K=-1 TO 1
630 AA=1#:BB=1#
640 FOR JJ=-1 TO 1
650 IF JJ<>K THEN AA=AA*(DP(I)-TAUL(1+JJ)):BB=BB*(TAUL(1+K)-TAUL(1+JJ))
660 NEXT JJ
670 DP3(I)=DP3(I)+DPL(1+K)*AA/BB
680 NEXT K
690 GOTO 1570
700 FOR K=-2 TO 0
710 AA=1#:BB=1#
720 FOR JJ=-2 TO 0
730 IF JJ<>K THEN AA=AA*(DP(I)-TAUL(II+JJ)):BB=BB*(TAUL(II+K)-TAUL(II+JJ))
740 NEXT JJ
750 DP3(I)=DP3(I)+DPL(II+K)*AA/BB
760 NEXT K
770 NEXT I
780 FOR I=1 TO 20
790 LGTAUU=LOG(DP3(I))/LN
800 FOR J=0 TO 41
810 IF LGTAUU<LGTAU(J) GOTO 1630
820 NEXT J
830 V=5#*(LGTAUU-LGTAU(J-1))-.5#:D1Y=TA(J)-TA(J-1):D2Y=TA(J-2)-TA(J-1)-TA(J)+TA(J+2)
840 D3Y=-TA(J-2)+3#*TA(J-1)-3#*TA(J)+TA(J+1)
850 TTA=(TA(J-1)+TA(J))/2#+V*D1Y+(V^2-.25#)*D2Y/4#+V*(V^2-.25#)*D3Y/6#
860 TTA=16349.64#/LOG(EXP(-TTA)+1#)
870 S(I)=1#/(EXP(16349.64#*8800#/LAMDA/TTA)-1#)
880 NEXT I
890 RETURN
900 *ABS.RATIO
910 REM Calculation of Absorption Coefficient Relative to Continuous Absorption Coefficient at 0.88 Micron
920 IF I0=1 THEN RR=1#:RETURN
930 RR=1#:I5=I4:I6=INUM+1
```

```
1740 WHILE ABSL(I5,1)<=LAMDA
1750 IF ABSL(I5,2)>=LAMDA GOTO 1770
1760 IF I5>=I6 GOTO 1780 ELSE I4=I5+1:GOTO 1780
1770 GOSUB *POPULATION:RR=RR+POP*ABSL(I5,3):I6=I5
1780 I5=I5+1:WEND
1790 RETURN
1800 *POPULATION
1810 AH=.01#:VH=3#*ABS(LAMDA-ABSL(I5,0))/(ABSL(I5,2)-ABSL(I5,1))
1820 GOSUB *VOIGT:POP=HAV
1830 RETURN
1840 *LINEWIDTH
1850 REM Determination of ABSL(N,1) AND ABSL(N,2) on the Basis of the Voigt Function
1860 VRAT=SQR(.016629#*TEMP/ATW+VMICRO^2)/299792#:J=0
1870 FOR I=1 TO INUM0
1880 IF ABSL(I,3)*RFAC<.001# GOTO 1990
1890 HAV=.001111#*ABSL(I,4)/ABSL(I,3)/RFAC:VH=1#/SQR((SQR(1#+6#*RP*HAV/AH)-1#)/3#)
1900 IF VH>=-.35#*LGAH+2.55# GOTO 1960
1910 HAV=.002#*ABSL(I,4)/ABSL(I,3)/RFAC+2#*AH/RP
1920 IF HAV>1# THEN VH=-.57#*LGAH+.53#:GOTO 1960
1930 VH=SQR(-LOG(HAV))
1940 IF VH<=-.57#*LGAH+.53# GOTO 1960
1950 VH=-.35#*LGAH+2.55#
1960 DLAM=VH*VRAT*ABSL(I,0):ABSL(I,1)=ABSL(I,0)-DLAM:ABSL(I,2)=ABSL(I,0)+DLAM
1970 IF ABSL(I,1)>LAMDAL OR ABSL(I,2)<LAMDAS GOTO 1990
1980 J=J+1:ABSL(J,0)=ABSL(I,0):ABSL(J,1)=ABSL(I,1):ABSL(J,2)=ABSL(I,2):ABSL(J,3)=ABSL(I,3)*RFAC/ABSL(I,4)
1990 NEXT I
2000 INUM=J:PRINT "Number of Lines = ";INUM;"    RFAC = ";RFAC:RETURN
2010 *SORT
2020 REM QUICK SORTING
2030 IPS=1:ILS=1:IUS=INUM:IEMPTY=0
2040 IS=ILS:JS=IUS:RKEY=ABSL((ILS+JS)¥2,1)
2050 WHILE RKEY>ABSL(IS,1)
2060 IS=IS+1
2070 WEND
2080 WHILE RKEY<ABSL(JS,1)
2090 JS=JS-1
2100 WEND
2110 IF IS<=JS THEN SWAP ABSL(IS,1),ABSL(JS,1):SWAP ABSL(IS,0),ABSL(JS,0):SWAP ABSL(IS,2),ABSL(JS,2)
2115 SWAP ABSL(IS,3),ABSL(JS,3):IS=IS+1:JS=JS-1:IF IS<=JS THEN 2050
2120 IF (IUS-IS)<(JS-ILS) THEN 2160
2130 IF IUS>IS THEN IAS=IS:IBS=IUS:GOSUB *PUSH
2140 IUS=JS
2150 GOTO 2180
2160 IF JS>ILS THEN IAS=ILS:IBS=JS:GOSUB *PUSH
2170 ILS=IS
2180 IF IUS>ILS THEN 2040
2190 GOSUB *POP
2200 IF NOT IEMPTY THEN 2040
2210 RETURN
2220 *PUSH
2230 IF IPS>13 THEN PRINT "STack Overflow":RETURN 2210
2240 ITS(IPS,1)=IAS:ITS(IPS,2)=IBS:IPS=IPS+1
2250 RETURN
2260 *POP
2270 IPS=IPS-1
2280 IF IPS=0 THEN IEMPTY=-1:RETURN
2290 ILS=ITS(IPS,1):IUS=ITS(IPS,2)
2300 RETURN
2310 *VOIGT
2320 REM Calculation of the Voigt Profile
2330 TH=1#/(4#*AH*AH):XH=VH/AH:Z1=XH*XH/4#/TH
2340 IF VH>4.2# GOTO 2470
2350 U(0)=RP/2#/SQR(TH):U(1)=-.5#/TH:SS(0)=U(0)+U(1)
2360 IH=2:U(2)=U(0)/2#/TH/IH
2370 WHILE ABS(U(IH))>.00001#
2380 SS(0)=SS(0)+U(IH):IH=IH+1:U(IH)=U(IH-2)/2#/TH/IH
2390 WEND
2400 IH=1:U0=SS(0):ZH=1#:ZZ=SS(0)
2410 WHILE ABS(ZZ)>.000001#
2420 SS(IH)=(1#-SS(IH-1))/(2#*IH-1#)/2#/TH:ZH=ZH*Z1/IH:ZZ=ZH*SS(IH)
2430 U0=U0+ZZ
2440 IH=IH+1:WEND
2450 U0=EXP(-Z1)*U0:HAV=U0/AH/RP
2460 RETURN
2470 IH=1:VV(0)=-1#/XH:VV(1)=-1#/XH/XH:U01=-VV(1)
2480 WHILE ABS(VV(2*IH-1))>.00001#
2490 IH=IH+1:VV(2*IH-2)=VV(2*IH-3)/XH-2#*TH*(2*IH-3)*VV(2*IH-4)/XH/XH
2495 VV(2*IH-1)=VV(2*IH-2)/XH-2#*TH*(2*IH-2)*VV(2*IH-3)/XH/XH
2500 U01=U01+(-1#)^IH*VV(2*IH-1)
2510 WEND
2520 HAV=U01/AH/RP
2530 RETURN
```

```
3000 DATA 5884.39#,0.0356#,0.68#,5884.52#,0.0272#,0.66#,5884.55#,0.0952#,0.68#,5884.61#,0.1679#,1.78#,5884.61#,0.1352#,1.74#
3010 DATA 5884.76#,0.1234#,0.76#,5884.77#,0.1509#,1.04#,5884.91#,0.704#,1.79#,5885.46#,0.1577#,1.74#,5885.46#,0.0768#,1.77#
3020 DATA 5885.71#,0.291#,1.81#,5885.89#,0.0181#,1.74#,5886.14#,0.0886#,0.74#,5886.27#,0.1044#,0.68#,5886.34#,0.033#,0.66#
3030 DATA 5886.44#,0.29#,1.72#,5886.45#,0.1808#,1.79#,5886.54#,0.1806#,1.75#,5886.62#,0.0032#,0.66#,5887.07#,0.0334#,0.71#
3040 DATA 5887.07#,0.0388#,1.74#,5887.07#,0.0826#,1.76#,5887.84#,0.2039#,1.75#,5887.86#,0.04#,0.68#,5887.86#,0.128#,0.77#
3050 DATA 5888.15#,0.1138#,0.69#,5888.26#,0.0098#,1.02#,5888.27#,0.0785#,1.78#,5888.41#,0.0386#,0.66#,5888.45#,0.0613#,1.81#
3060 DATA 5888.48#,0.0563#,1.74#,5888.48#,0.1939#,1.8#,5888.63#,0.0936#,0.75#,5889.37#,0.2277#,1.76#,5889.38#,0.0047#,0.66#
3070 DATA 5889.57#,0.0043#,1.74#,5890.17#,0.0726#,1.74#,5890.25#,0.035#,0.71#,5890.38#,0.0938#,1.76#,5890.7#,0.2071#,1.8#
3080 DATA 5890.71#,0.0027#,0.84#,5890.99#,0.0626#,0.41#,5891.14#,0.2518#,1.76#,5891.16#,0.1327#,0.78#,5891.27#,0.0986#,0.76#
3090 DATA 5891.27#,0.0787#,1.78#,5891.45#,0.0871#,0.76#,5891.45#,0.3206#,1.72#,5891.75#,0.3161#,1.83#,5892.02#,0.0086#,1.74#
3100 DATA 5892.02#,0.0881#,1.74#,5892.24#,0.006#,0.66#,5892.63#,0.7227#,1.81#,5893.11#,0.2762#,1.76#,5893.38#,0.1326#,0.7#
3110 DATA 5893.63#,0.0275#,0.72#,5893.91#,0.1053#,1.76#,5894.1#,0.1037#,0.77#,5894.11#,0.103#,1.75#,5894.5#,0.0953#,1.79#
3120 DATA 5894.63#,0.2994#,1.74#,5894.64#,0.0126#,1.74#,5894.64#,0.1374#,0.79#,5895.07#,0.3286#,1.84#,5895.25#,0.049#,0.69#
3130 DATA 5895.27#,0.3009#,1.77#,5895.32#,0.0071#,0.66#,5895.71#,0.0685#,0.93#,5895.86#,0.2335#,1.82#,5895.86#,0.0543#,0.68#
3140 DATA 5895.94#,0.1422#,0.71#,5895.94#,0.0093#,1.04#,5896.44#,0.1174#,1.75#,5896.55#,0.0459#,1.82#,5897.17#,0.027#,0.73#
3150 DATA 5897.17#,0.1087#,0.78#,5897.62#,0.1604#,1.74#,5897.62#,0.1171#,1.77#,5897.71#,0.3257#,1.78#,5897.91#,0.077#,1.8#
3160 DATA 5898.75#,0.2468#,1.83#,5898.75#,0.008#,0.67#,5898.75#,0.0593#,0.68#,5898.75#,0.1518#,0.71#,5898.91#,0.0646#,0.95#
3170 DATA 5899.01#,0.1314#,1.76#,5899.2#,0.0536#,0.7#,5899.85#,0.1587#,1.08#,5900.25#,0.1279#,0.87#,5900.34#,0.3508#,1.78#
3180 DATA 5900.67#,0.7414#,1.82#,5900.7#,0.019#,1.75#,5900.9#,0.0194#,0.74#,5901.51#,0.1291#,1.78#,5901.51#,0.0761#,1.8#
```